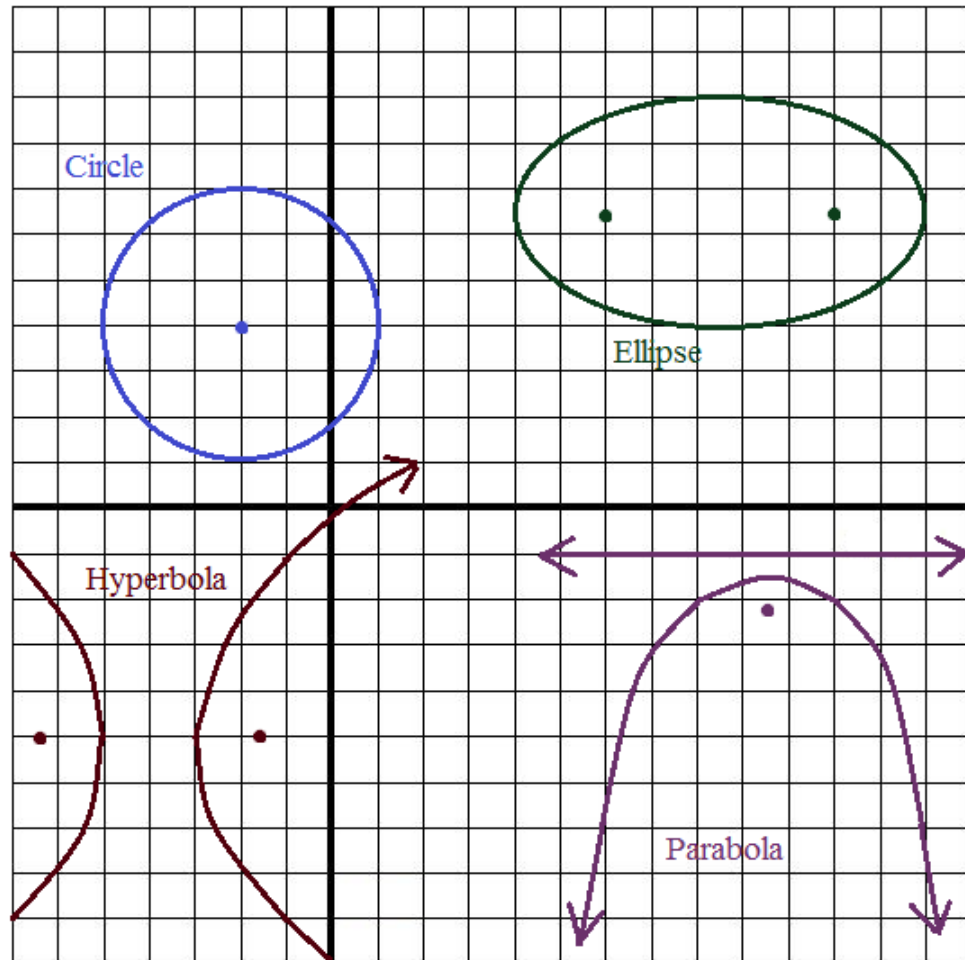


Complete Conics:

Notes, Examples, and Graphs



****Includes 4 Practice Tests (& Solutions)**

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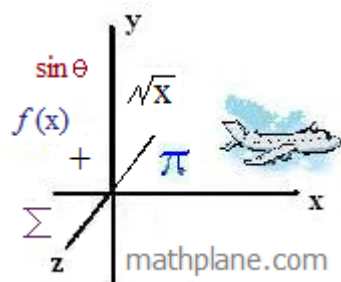
Examples, vertical/horizontal, sketching

Practice Quiz (& Solutions)

Thanks for checking out this math packet. Inside are conics notes, formulas, graphs, and more. The examples and practice tests are designed to help you learn/review this important Algebra II topic.

If you have any feedback, questions, or suggestions, let us know!

Lance...

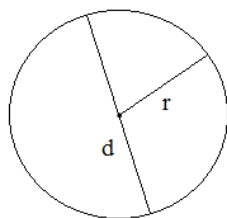


Circle



Circle

What is it? A set of (coplaner) points equidistant from a given point (the center).



r = radius
 d = diameter

$$d = 2r$$

$$\text{Area} = \pi r^2$$

$$\text{Circumference} = 2\pi r \text{ or } \pi d$$

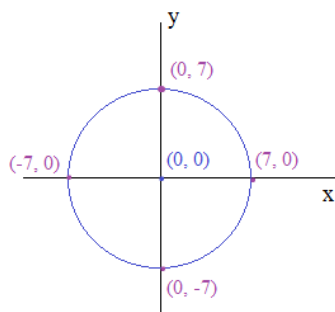
General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
 where $A = C$

Standard Form: $(x-h)^2 + (y-k)^2 = r^2$
 where (h, k) is the center
 r is the radius

Example 1: $x^2 + y^2 = 49$

$(h, k) = (0, 0)$
 The center is the origin!

$r^2 = 49$
 The radius is 7



Comments:

Standard form: $x^2 + y^2 - 49 = 0$

The coefficients of x^2 and y^2 are equal (both are 1)

Any point on the circle will satisfy the equation. $(0)^2 + (7)^2 = 49$ $(-7)^2 + (0)^2 = 49$

The distance from any point to the center = 7

Example 2: $(x+2)^2 + (y-3)^2 = 25$

$h = -2$
 $k = 3$

The center is $(-2, 3)$

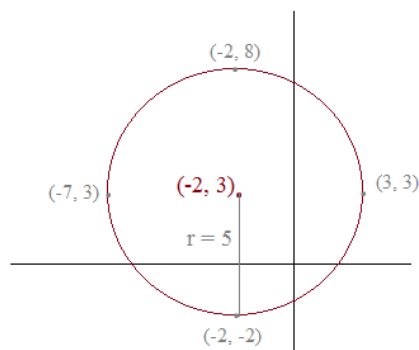
The radius is 5

Comments:

Because the standard form is $(x-h)^2 + (y-k)^2 = r^2$
 the center is $(-2, 3)$, NOT $(2, -3)$

Since the center is $(-2, 3)$ and the radius is 5,
 we know the following points are on the circle:

$(-7, 3)$ 5 units left
 $(3, 3)$ 5 units right
 $(-2, 8)$ 5 units up
 $(-2, -2)$ 5 units down



To check the graph, plug a few points into the equation:

$(3, 3)$ $((3) + 2)^2 + ((3) - 3)^2 = 25$
 $5^2 + 0^2 = 25$ ✓

$(-2, 8)$ $((-2) + 2)^2 + ((8) - 3)^2 = 25$
 $0^2 + 5^2 = 25$ ✓

Circles, Completing the Square & Inequalities

Example 3: $x^2 + y^2 - 4x = 21$

The coefficients of x^2 and y^2 are the same (1), so we know it's a circle.

To find the center, we complete the square. Then, express the equation in standard form.

$$x^2 + y^2 - 4x = 21$$

$$x^2 - 4x + y^2 = 21$$

$$x^2 - 4x + 4 + y^2 = 21 + 4$$

$$(x - 2)(x - 2) + y^2 = 25$$

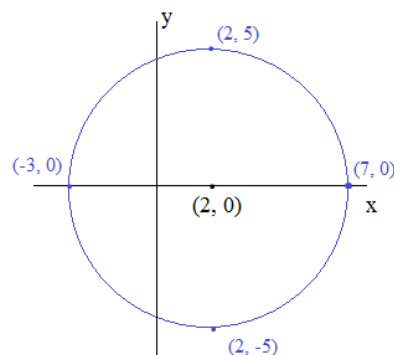
$$(x - 2)^2 + (y - 0)^2 = 25$$

$$h = 2$$

$$k = 0$$

The center is (2, 0)

The radius is 5



To check your solutions, plug points into original equation:

$$(2, 5): (2)^2 + (5)^2 - 4(2) = 21$$

$$4 + 25 - 8 = 21 \quad \checkmark$$

$$(-3, 0): (-3)^2 + (0)^2 - 4(-3) = 21$$

$$9 + 0 + 12 = 21 \quad \checkmark$$

Example 4: $x^2 + y^2 + 6x - 16y + 48 \leq 0$

At first, we'll ignore the $<$ sign and rewrite the equation in standard form.

Step 1: Complete the square:

$$x^2 + 6x + y^2 - 16y = -48$$

$$x^2 + 6x + 9 + y^2 - 16y + 64 = -48 + 9 + 64$$

$$(x + 3)(x + 3) + (y - 8)(y - 8) = 25$$

$$(x + 3)^2 + (y - 8)^2 = 25$$

The circle has center (-3, 8) and radius 5

Step 2: Graph and verify circle

$$x^2 + y^2 + 6x - 16y + 48 = 0$$

$$(2, 8): (2)^2 + (8)^2 + 6(2) - 16(8) + 48 = 0$$

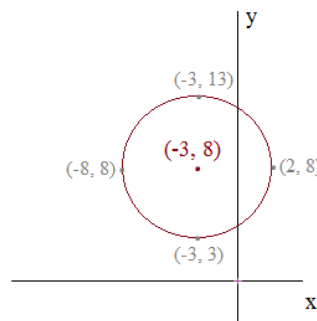
$$4 + 64 + 12 - 128 + 48 = 0$$

$$0 = 0 \quad \checkmark$$

$$(-3, 13): (-3)^2 + (13)^2 + 6(-3) - 16(13) + 48 = 0$$

$$9 + 169 - 18 - 208 + 48 = 0$$

$$0 = 0 \quad \checkmark$$



Step 3: Test points in original equation to determine the shaded area.

$$x^2 + y^2 + 6x - 16y + 48 \leq 0$$

(0, 0) is outside the circle

(-3, 8) is inside the circle

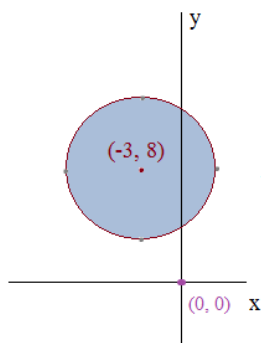
$$\text{Test } (0, 0): (0)^2 + (0)^2 + 6(0) - 16(0) + 48 \leq 0$$

$$0 + 0 + 0 - 0 + 48 \leq 0 \quad \text{NO}$$

$$\text{Test } (-3, 8): (-3)^2 + (8)^2 + 6(-3) - 16(8) + 48 \leq 0$$

$$9 + 64 - 18 - 128 + 48 \leq 0 \quad \text{YES!}$$

Shaded region will include (-3, 8) and exclude (0, 0)



Find the equation of a line tangent to $x^2 + y^2 - 10x - 8y = -16$ at the point $(2, 8)$

Step 1: Identify the Conic

Complete the square to put into Standard Form.

$$x^2 + y^2 - 10x - 8y = -16$$

$$x^2 - 10x + 25 + y^2 - 8y + 16 = -16 + 25 + 16$$

$$(x - 5)^2 + (y - 4)^2 = 25$$

$$(x - 5)^2 + (y - 4)^2 = 25$$

Circle with center $(5, 4)$ and radius 5

Step 2: Sketch a graph and verify.

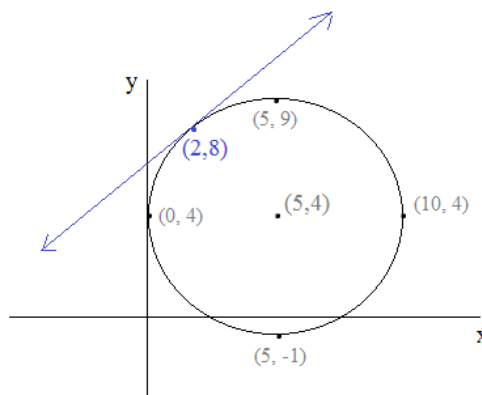
The center is $(5, 4)$... And, the radius is 5 ..

So, $(5, 9)$ up
 $(10, 4)$ right
 $(5, -1)$ down
 $(0, 4)$ left
are all 5 units from the center

and, using the distance formula:

$$\sqrt{(5 - 2)^2 + (4 - 8)^2} = 5$$

we can verify that $(2, 8)$ is 5 units from the center.



Note: To check your work, substitute a few points into the equation.

$$(0, 4): (0)^2 + (4)^2 - 10(0) - 8(4) = -16$$

$$0 + 16 - 0 - 32 = -16 \quad \checkmark$$

$$(10, 4): (10)^2 + (4)^2 - 10(10) - 8(4) = -16$$

$$100 + 16 - 100 - 32 = -16 \quad \checkmark$$

$$(2, 8): (2)^2 + (8)^2 - 10(2) - 8(8) = -16$$

$$4 + 64 - 20 - 64 = -16 \quad \checkmark$$

All these points fit into the original equation; Therefore, they are all points on the circle!

Step 3: Identify the tangent line

To find the equation of a line, you need a point and the slope.

Point: $(2, 8)$

What is the slope? Since a line tangent to a circle is perpendicular to the adjoining radius, we need to find the slope of that radius!

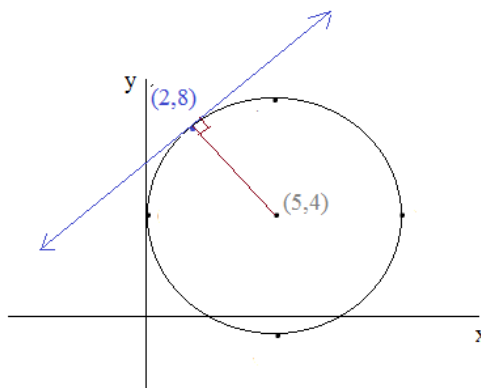
slope of segment connecting $(2, 8)$ and $(5, 4)$

$$m = \frac{8 - 4}{2 - 5} = -\frac{4}{3}$$

Therefore, the slope of a perpendicular line is $\frac{3}{4}$

So, the equation of a line tangent to the circle at $(2, 8)$ is

$$(y - 8) = \frac{3}{4}(x - 2)$$



Circles Practice Quiz

I. Parts of a circle

Identify the radius, center, and 3 points on each circle:

A) $x^2 + y^2 = 9$

center: _____

radius: _____

3 points: _____

B) $x^2 + (y - 3)^2 = 4$

center: _____

radius: _____

3 points: _____

C) $(x - 2)^2 + (y + 1)^2 = 3$

center: _____

radius: _____

3 points: _____

D) $x^2 + 10x + y^2 = -24$

center: _____

radius: _____

3 points: _____

E) $3x^2 + 3y^2 + 6x + 12y + 9 = 0$

center: _____

radius: _____

3 points: _____

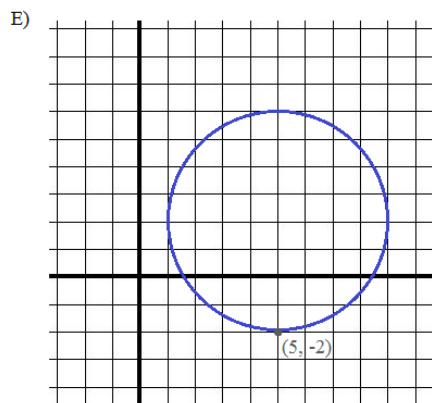
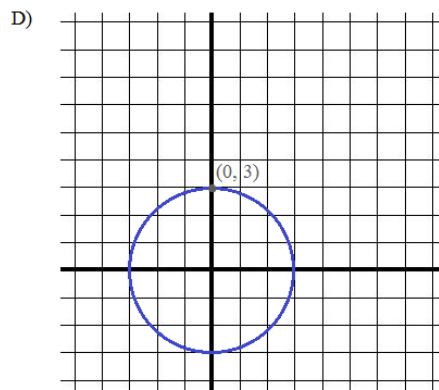
II. Equation of a Circle

Determine the equation of each circle from the given information.
(Express your answer in both *standard* form and *general* form)

A) radius: 4
center: origin

B) radius: 5
center: (2, 4)

C) radius: 7
center: (-4, 0)

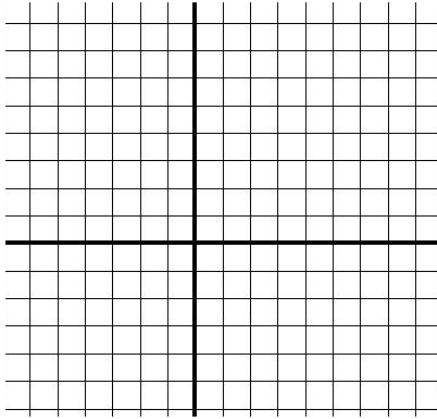


Circles Practice Quiz

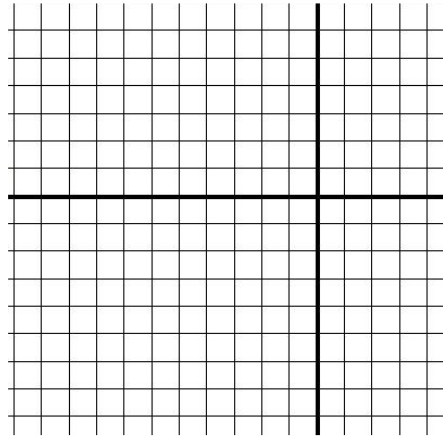
III. Graphing circles

Graph the following circles; label the center, radius, and 2 points

A) $x^2 + (y - 4)^2 = 16$



B) $x^2 + 6x + y^2 + 2y - 15 = 0$

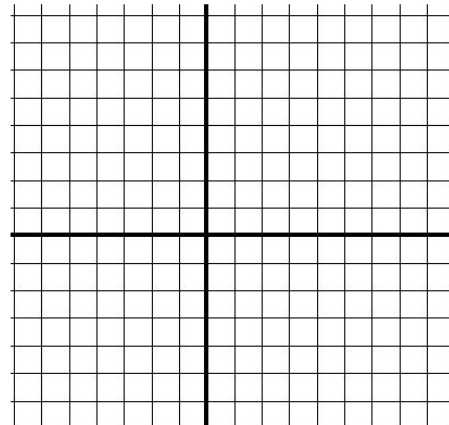


IV. Applications

- A) The diameter of a circle has endpoints $(2, 4)$ and $(8, 4)$.
What is the equation of the circle?

- B) Given: circle $x^2 + y^2 = 25$
line $x + y = 3$

What are the points of intersection?
Solve algebraically and verify graphically.



- C) What is the equation of the line *tangent* to
the circle $(x - 3)^2 + (y - 4)^2 = 10$
through the point $(4, 1)$?
What is the equation of the *normal* through $(4, 1)$?

Circles Practice Quiz

SOLUTIONS

$$\text{Standard form of circle: } (x - h)^2 + (y - k)^2 = r^2$$

where (h, k) is the vertex
 r is the radius

I. Parts of a circle

Identify the radius, center, and 3 points on each circle:

A) $x^2 + y^2 = 9$

center: $(0, 0)$

radius: 3

3 points: $(0, 3)$

$(3, 0)$

$(-3, 0)$

(note: there are more than 3 possible points; any (x, y) that is a solution is a point)

B) $x^2 + (y - 3)^2 = 4$

center: $(0, 3)$

radius: 2

3 points: $(0, 1)$

$(2, 3)$

$(0, 5)$

C) $(x - 2)^2 + (y + 1)^2 = 3$

center: $(2, -1)$

radius: $\sqrt{3}$

3 points: $(\sqrt{3} + 2, -1)$

$(-\sqrt{3} + 2, -1)$

$(2, \sqrt{3} - 1)$

D) $x^2 + 10x + y^2 = -24$

complete the square to put into standard form:

center: $(-5, 0)$

radius: 1

3 points: $(-5, 1)$

$(-5, -1)$

$(-4, 0)$

$x^2 + 10x + y^2 = -24$

$x^2 + 10x + 25 + y^2 = -24 + 25$

$(x + 5)(x + 5) + y^2 = 1$

$(x + 5)^2 + y^2 = 1$

E) $3x^2 + 3y^2 + 6x + 12y + 9 = 0$

center: $(-1, -2)$

radius: $\sqrt{2}$

3 points: $(0, -1)$

$(-2, -1)$

$(0, -3)$

$3x^2 + 3y^2 + 6x + 12y + 9 = 0$ (divide by 3 and rearrange)

$x^2 + 2x + y^2 + 4y + 3 = 0$

$x^2 + 2x + 1 + y^2 + 4y + 4 = -3 + 1 + 4$

$(x + 1)(x + 1) + (y + 2)(y + 2) = 2$

$(x + 1)^2 + (y + 2)^2 = 2$

II. Equation of a Circle

Determine the equation of each circle from the given information.
(Express your answer in both *standard form* and *general form*)

A) radius: 4
center: origin

Standard Form: $(x - 0)^2 + (y - 0)^2 = 16$

$x^2 + y^2 = 16$

General Form: $x^2 + y^2 - 16 = 0$

B) radius: 5
center: $(2, 4)$

Standard Form: $(x - 2)^2 + (y - 4)^2 = 25$

(expand terms and simplify)

General Form: $x^2 - 4x + 4 + y^2 - 8y + 16 = 25$

$x^2 + y^2 - 4x - 8y - 5 = 0$

C) radius: 7
center: $(-4, 0)$

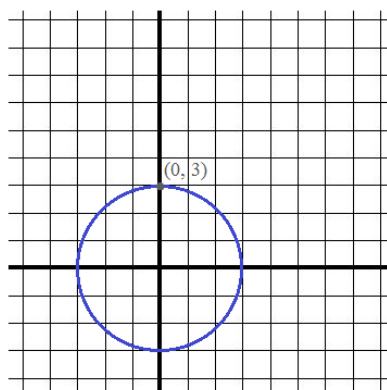
Standard Form: $(x + 4)^2 + y^2 = 49$

General Form: $x^2 + 8x + 16 + y^2 - 49 = 0$

$x^2 + 8x + y^2 - 33 = 0$

$x^2 + y^2 + 8x - 33 = 0$

D)



Center: $(0, 0)$
Radius: 3

Standard Form:

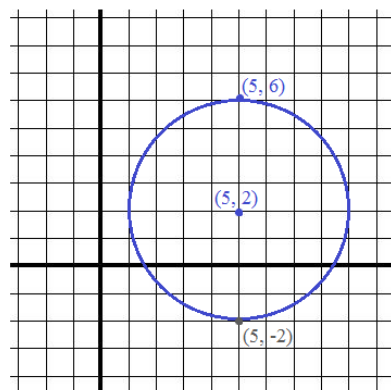
$x^2 + y^2 = 9$

General Form:

$x^2 + y^2 - 9 = 0$

(Note: you may test points to check your equation)

E)



Center: $(5, 2)$
Radius: 4

Standard Form:

$(x - 5)^2 + (y - 2)^2 = 16$

General Form:

$x^2 - 10x + 25 + y^2 - 4y + 4 = 16$

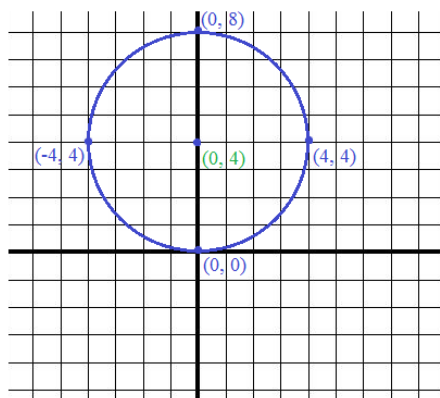
$x^2 + y^2 - 10x - 4y + 13 = 0$

SOLUTIONS

III. Graphing circles

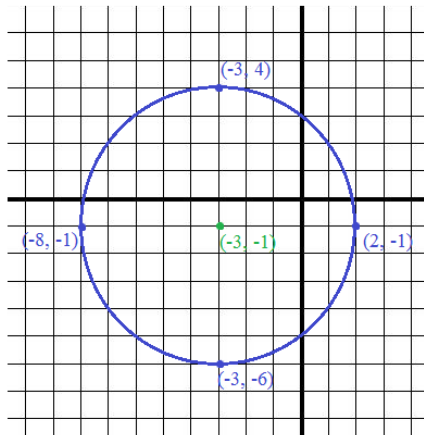
Graph the following circles; label the center, radius, and 2 points

A) $x^2 + (y - 4)^2 = 16$



Center: $(0, 4)$
Radius: 4
 $\sqrt{16}$

B) $x^2 + 6x + y^2 + 2y - 15 = 0$



Change to Standard form
(to show center and radius):

$$\begin{aligned} x^2 + 6x + y^2 + 2y - 15 &= 0 \\ x^2 + 6x + 9 + y^2 + 2y + 1 &= 15 + 9 + 1 \\ (x + 3)^2 + (y + 1)^2 &= 25 \end{aligned}$$

Center: $(-3, -1)$
Radius: 5

IV. Applications

- A) The diameter of a circle has endpoints $(2, 4)$ and $(8, 4)$.
What is the equation of the circle?

Since the endpoints of the diameter are given, we
can use the midpoint to find the center of the circle:

center: $(5, 4)$

Since the distance between endpoints is 6, the radius is 3

$$(x - 5)^2 + (y - 4)^2 = 9$$

- B) Given: circle $x^2 + y^2 = 25$
line $x + y = 3$

What are the points of intersection?

Solve algebraically and verify graphically.

Algebraically, we'll use substitution: $x = 3 - y$

$$(3 - y)^2 + y^2 = 25$$

$$9 - 6y + y^2 + y^2 = 25$$

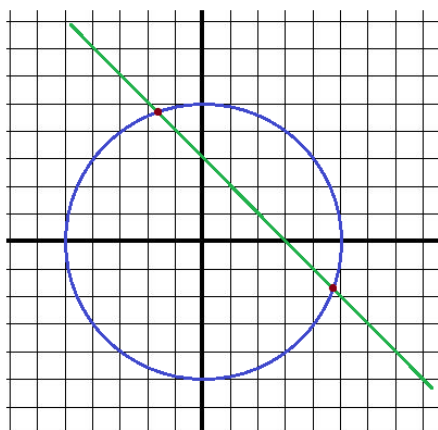
$$2y^2 - 6y - 16 = 0$$

$$y^2 - 3y - 8 = 0$$

$$y = \frac{3 \pm \sqrt{9 - 4(1)(-8)}}{2(1)} = \frac{3 \pm \sqrt{41}}{2} \text{ and } \frac{3 - \sqrt{41}}{2}$$

$(-1.7, 4.7)$ $(4.7, -1.7)$
approximate pts. of intersection

(substitute each y
solution to find x)



- C) What is the equation of the line *tangent* to
the circle $(x - 3)^2 + (y - 4)^2 = 10$

through the point $(4, 1)$?

What is the equation of the *normal* through $(4, 1)$?

(To find the equation of a line, we need a point and slope)

Point: $(4, 1)$

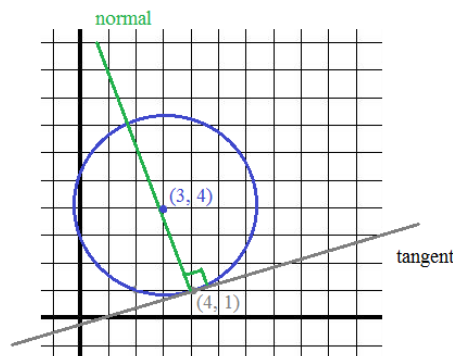
Slope? (**A tangent line is perpendicular to the
normal; and, the normal goes through the center!)

slope of line through $(3, 4)$ and $(4, 1)$ is -3

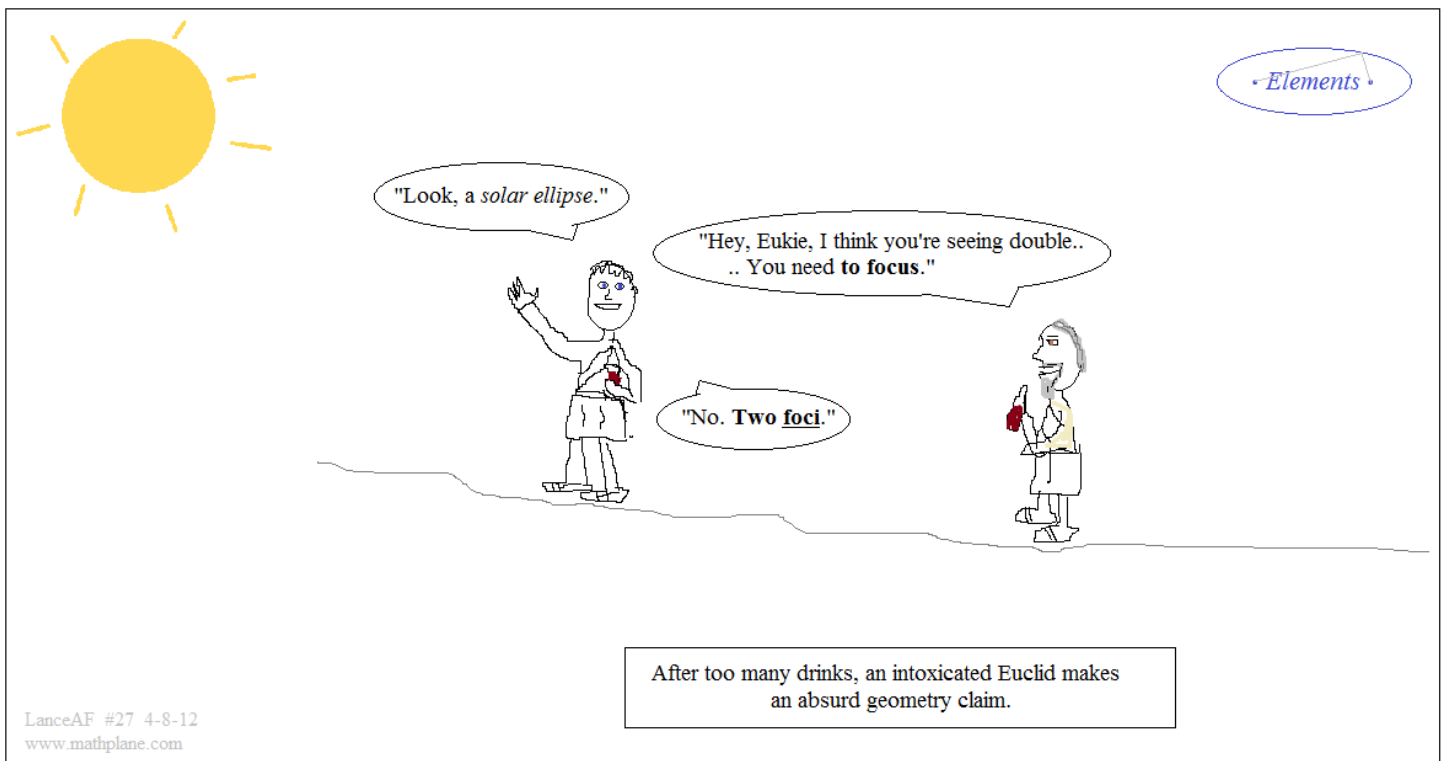
Equation of normal: $(y - 1) = -3(x - 4)$

Equation of tangent: $(y - 1) = \frac{1}{3}(x - 4)$

(slope of perpendicular line:
opposite reciprocal $\rightarrow 1/3$)



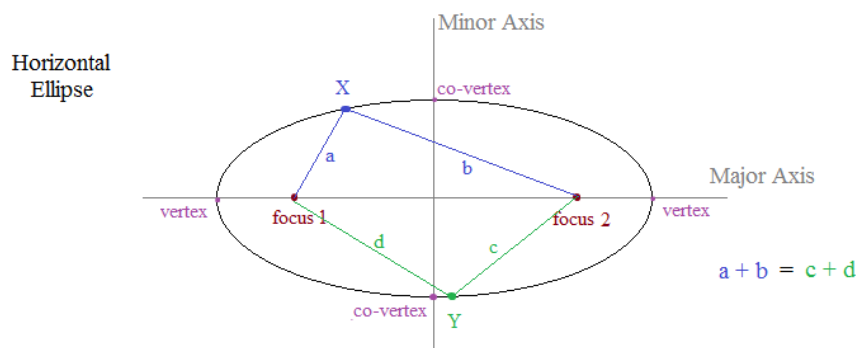
ELLIPSE



Ellipse

What is it? A "stretched circle".

A set of (coplanar) points that have the same 'sum of distances to 2 given points (foci)'.



General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

where $A \neq C$

and

A and C have the same sign
(neither can be 0)

Standard Form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

where (h, k) is the center

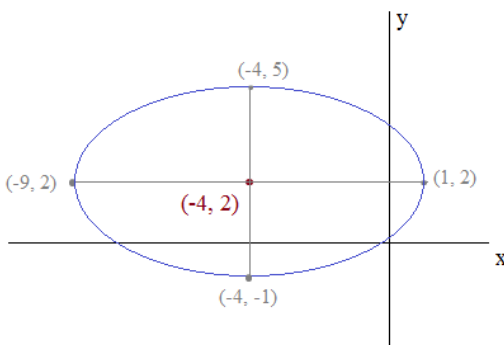
a is the "horizontal radius" (distance from center to 'horizontal end')

b is the "vertical radius" (distance from center to 'vertical end')

Example 1: $\frac{(x+4)^2}{25} + \frac{(y-2)^2}{9} = 1$

$h = -4$
 $k = 2$ center is $(-4, 2)$

$a = 5$ major axis has length 10 units
 $b = 3$ minor axis has length 6 units



Comments:

If $a > b$, then the ellipse will be horizontal. (major axis is parallel to x-axis)

If $a < b$, then the ellipse will be vertical. (major axis is parallel to y-axis)

$a \neq b$. (If $a = b$, then it would be a circle!)

Any point on the ellipse will satisfy the equation.

'major vertex' $(1, 2)$: $\frac{((1)+4)^2}{25} + \frac{((2)-2)^2}{9} = 1$ $\frac{25}{25} + \frac{0}{9} = 1$

'minor/co-vertex' $(-4, 5)$: $\frac{((-4)+4)^2}{25} + \frac{((5)-2)^2}{9} = 1$ $\frac{0}{25} + \frac{9}{9} = 1$

If c = distance from center to each focus.

$2a$ = length of horizontal axis

$2b$ = length of vertical axis

then $c^2 = a^2 - b^2$ when $a > b$ (horizontal ellipse)

$c^2 = b^2 - a^2$ when $b > a$ (vertical ellipse)

$$\frac{(x+4)^2}{25} + \frac{(y-2)^2}{9} = 1$$

$$a = 5$$

(length of horizontal axis = 10)

$$b = 3$$

(length of vertical axis = 6)

Since $a > b$, it's a horizontal ellipse.

So, to find the foci, we'll use

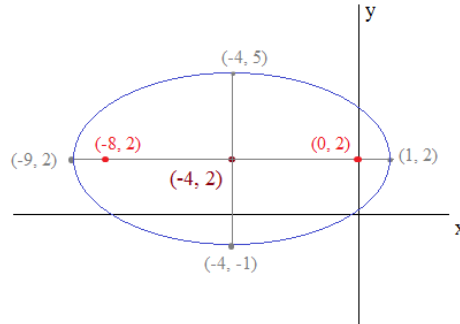
$$c^2 = a^2 - b^2$$

$$c^2 = 25 - 9$$

$$c = \pm 4$$

Since the center is $(-4, 2)$,

the foci are $(-8, 2)$ and $(0, 2)$



Example 2:

$$4x^2 + y^2 - 40x + 6y + 93 = 0$$

To find the center, we complete the square. Then, express the equation in standard form.

$$4x^2 + y^2 - 40x + 6y + 93 = 0$$

$$4x^2 - 40x + y^2 + 6y = -93$$

$$4(x^2 - 10x + 25) + y^2 + 6y + 9 = -93 + 4(25) + 9$$

$$4(x-5)(x-5) + (y+3)(y+3) = 16$$

$$4(x-5)^2 + (y+3)^2 = 16$$

$$\frac{(x-5)^2}{4} + \frac{(y+3)^2}{16} = 1$$

center: $(5, -3)$

$$a = 2$$

'minor' vertices: $(3, -3)$ $(7, -3)$

$$b = 4$$

'major' vertices: $(5, 1)$ $(5, -7)$

Since $b > a$, it's a vertical ellipse.

(major axis is parallel to y-axis)

To find foci, use $c^2 = b^2 - a^2$

$$= 16 - 4 = 12$$

$$c = \pm 2\sqrt{3}$$

foci are $(5, -3 + 2\sqrt{3})$ and $(5, -3 - 2\sqrt{3})$

Comments: The distance from focus to minor vertex $(7, -3) = 4$
and the distance from center to major vertex $(5, 1) = 4$

To verify graph, plug points into original equation.

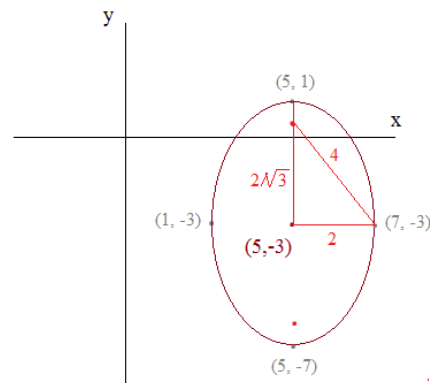
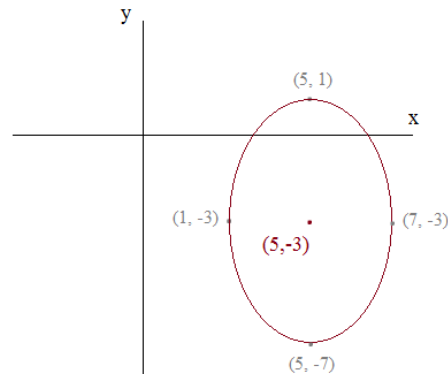
$$4x^2 + y^2 - 40x + 6y + 93 = 0$$

$$(5, 1): 4(5)^2 + (1)^2 - 40(5) + 6(1) + 93 = 0$$

$$100 + 1 - 200 + 6 + 93 = 0$$

$$0 = 0$$

The coefficients of x^2 and y^2 are 4 and 1.
Since they are different, the same sign, and
neither is zero, it is an ellipse.



Ellipses: constructing and solving equations

Example 3: Given Foci: (2, 1) and (8, 1)
Length of major axis: 8 units

- Write equations of the ellipse in Standard and General Forms
- Graph the Ellipse
- Where does the line $x = 3$ intersect this ellipse?

Step 1: Identify given parts

Foci: (2, 1) and (8, 1)

Since the midpoint is (5, 1), we know the center is (5, 1)

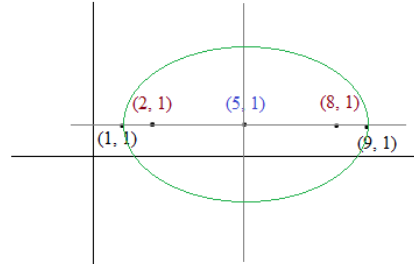
$$\begin{aligned} h &= 5 \\ k &= 1 \end{aligned}$$

and the minor axis is a vertical segment through (5, 1)

Length of the major axis is 8 units. Since the major axis will pass through the foci and center, we know it's a horizontal line segment with midpoint (5, 1).

Therefore, the major vertices are (1, 1) and (9, 1).

(4 units left of center & 4 units right of center)

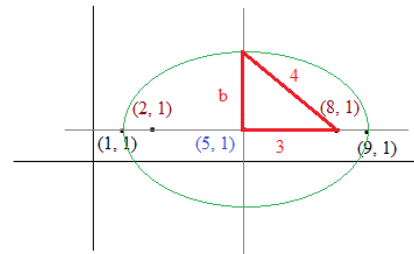


Step 2: Find length of minor axis.

For a horizontal ellipse, $c^2 = a^2 - b^2$ (because $a > b$)

$$\begin{aligned} \text{(center to focus)} \quad c &= 3 \\ \text{(length of major semi-axis)} \quad a &= 4 \\ \text{(length of minor semi-axis)} \quad b &= ? \end{aligned}$$

$$\begin{aligned} (3)^2 &= (4)^2 - b^2 \\ 9 &= 16 - b^2 \\ b &= \sqrt{7} \end{aligned}$$



Step 3: Construct the Equations!

Standard Form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$$\frac{(x-5)^2}{16} + \frac{(y-1)^2}{7} = 1$$

(Use algebra to convert to General Form)

$$7(x-5)^2 + 16(y-1)^2 = 112$$

$$7x^2 - 70x + 175 + 16y^2 - 32y + 16 = 112$$

$$7x^2 + 16y^2 - 70x - 32y + 79 = 0$$

center: (5, 1)
major axis (2a) : 8 units
minor axis (2b) : $2\sqrt{7}$ units

$$\begin{aligned} h &= 5 \\ k &= 1 \\ a &= 4 \\ b &= \sqrt{7} \end{aligned}$$

Where does $x = 3$ intersect the ellipse?

(plug $x = 3$ into one of the equations above)

$$\frac{(3-5)^2}{16} + \frac{(y-1)^2}{7} = 1$$

$$\frac{4}{16} + \frac{(y-1)^2}{7} = 1$$

$$\frac{(y-1)^2}{7} = \frac{3}{4}$$

$$\frac{21}{4} = (y-1)^2$$

$$\pm \frac{\sqrt{21}}{2} = y - 1$$

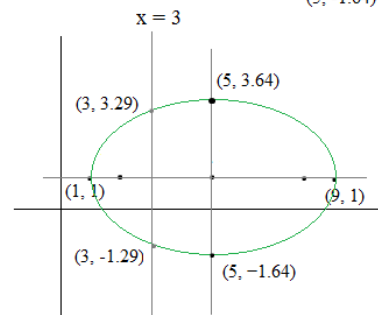
$$y = 3.29 \text{ and } -1.29$$

Intersection points:

$$\begin{aligned} (3, 3.29) \\ \text{and} \\ (3, -1.29) \end{aligned}$$

(minor vertices)

$$\begin{aligned} (5, 3.64) &= (5, 1 + \sqrt{7}) \\ (5, -1.64) &= (5, 1 - \sqrt{7}) \end{aligned}$$



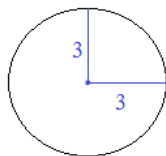
Eccentricity (conics)

What is it? A number that expresses how drawn out ("stretch" or "ovally") a conic section is...

Examples:

Circle: $e = 0$

Ellipse: $0 < e < 1$

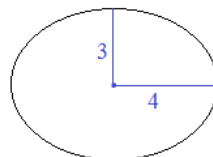


$$x^2 + y^2 = 9$$

(note: the vertical distance from the center to the circle is equal to the horizontal distance --- 3)

$$e = 0$$

(one could say $a = 3$
 $b = 3$
 $c = 0$)



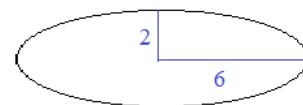
$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$e = \frac{c}{a} = \frac{\sqrt{7}}{4} \approx .66$$

$$a = 4$$

$$b = 3$$

$$c = \sqrt{7}$$



$$\frac{x^2}{36} + \frac{y^2}{4} = 1$$

$$e = \frac{c}{a} = \frac{\sqrt{32}}{6} \approx .94$$

$$a = 6$$

$$b = 2$$

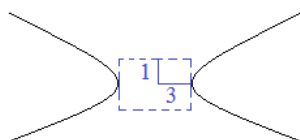
$$c = 4\sqrt{2}$$

Hyperbolas and Parabolas have eccentricity, too...

Hyperbola: $e > 1$

Parabola: $e = 1$

Examples:



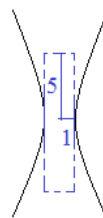
$$\frac{x^2}{9} - \frac{y^2}{1} = 1$$

$$e = \frac{\sqrt{10}}{3} \approx 1.05$$

$$a = 3$$

$$b = 1$$

$$c = \sqrt{10}$$



$$\frac{x^2}{1} - \frac{y^2}{25} = 1$$

$$e = \frac{\sqrt{26}}{1} \approx 5.1$$

$$a = 1$$

$$b = 5$$

$$c = \sqrt{26}$$



$$x = \frac{1}{2}y^2$$

$$e = 1$$

one may say the 2 parts of the hyperbola are on top of each other

(i.e. the dimensions of the box are $a \times 0$)

horizontal ellipses and hyperbolas:

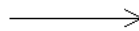
$$e = \frac{c}{a}$$

vertical ellipses and hyperbolas:

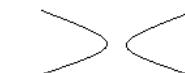
$$e = \frac{c}{b}$$



$$e = 0$$



$$e = 1$$



Circle and Ellipse:

System of Conics

Solve and Graph:

$$x^2 + 5y^2 = 49$$

$$x^2 + y^2 - 4x = 21$$

Solve (using substitution)

(1st equation)

$$x^2 + 5y^2 = 49$$

$$5y^2 = 49 - x^2$$

$$y^2 = \frac{49 - x^2}{5}$$

(2nd equation)

$$x^2 + y^2 - 4x = 21$$

("substitute 1st into 2nd")

$$x^2 + \frac{49 - x^2}{5} - 4x = 21$$

(multiply by 5)

$$5x^2 + 49 - x^2 - 20x = 105$$

(collect terms)

$$4x^2 - 20x - 56 = 0$$

(divide by 4)

$$x^2 - 5x - 14 = 0$$

(factor)

$$(x - 7)(x + 2) = 0$$

$$x = 7, -2$$

(plug x = 7 into 1st equation)

$$(7)^2 + 5y^2 = 49$$

$$49 + 5y^2 = 49$$

$$y = 0$$

(7, 0)

(check 2nd equation)

$$(7)^2 + (0)^2 - 4(7) = 21$$

$$49 + 0 - 28 = 21$$

$$21 = 21 \checkmark$$

(plug x = -2 into 1st equation)

$$(-2)^2 + 5y^2 = 49$$

$$4 + 5y^2 = 49$$

$$5y^2 = 45$$

$$y^2 = 9$$

$$y = \pm 3$$

(-2, 3)

(-2, -3)

(check 2nd equation)

$$(-2)^2 + (3)^2 - 4(-2) = 21$$

$$4 + 9 - (-8) = 21$$

$$21 = 21 \checkmark$$

$$(-2)^2 + (-3)^2 - 4(-2) = 21$$

$$4 + 9 - (-8) = 21$$

$$21 = 21 \checkmark$$

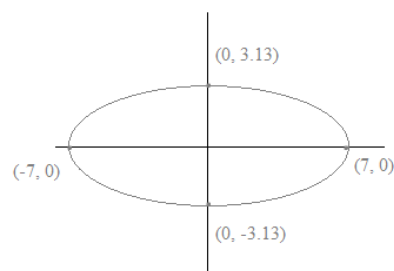
Graph 1st equation

$$x^2 + 5y^2 = 49$$

$$\frac{x^2}{49} + \frac{5y^2}{49} = 1$$

$$\frac{x^2}{49} + \frac{y^2}{49/5} = 1$$

Ellipse



Graph 2nd equation

$$x^2 + y^2 - 4x = 21$$

(complete the square)

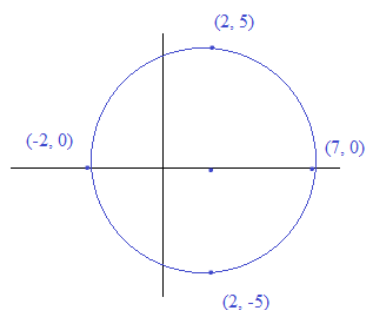
$$x^2 - 4x + y^2 = 21$$

$$x^2 - 4x + 4 + y^2 = 21 + 4$$

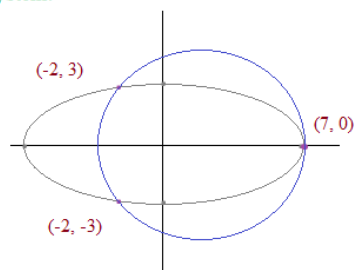
$$(x - 2)^2 + y^2 = 25$$

Circle

Center: 2, 0
Radius: 5



Graph the System!



Ellipses Practice Quiz

I. Parts of an Ellipse

↙ (optional)

Identify the center, vertices, co-vertices, foci, and 1 random point on each ellipse:

A) $\frac{(x+1)^2}{4} + \frac{(y+4)^2}{25} = 1$

center: _____

vertices: _____

co-vertices: _____

foci: _____

random point: _____

B) $4x^2 + 9y^2 = 36$

center: _____

vertices: _____

co-vertices: _____

foci: _____

random point: _____

C) $\frac{x^2}{5} + \frac{4(y-3)^2}{9} = 1$

center: _____

vertices: _____

co-vertices: _____

foci: _____

random point: _____

D) $3x^2 + 4y^2 - 18x + 8y + 19 = 0$

center: _____

vertices: _____

co-vertices: _____

foci: _____

random point: _____

E) $x^2 + 8x + 3y^2 - 12y = -1$

center: _____

vertices: _____

co-vertices: _____

foci: _____

random point: _____

II. Equation of an Ellipse

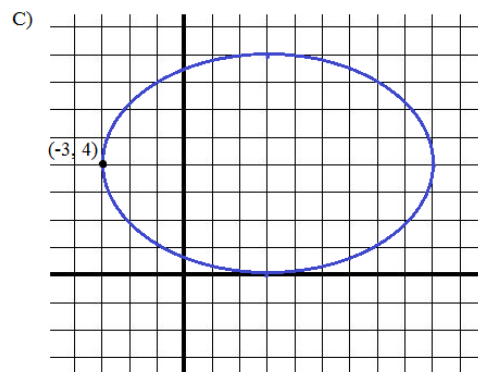
Determine the equation of each ellipse from the given information.

(Express your answer in both *standard* form and *general* form)

A) Foci: $(2, 7 + \sqrt{5})$
 $(2, 7 - \sqrt{5})$

One Vertex: $(0, 7)$

B) Major Intercepts: $(0, 6)$ $(0, -6)$
 Minor Intercepts: $(3, 0)$ $(-3, 0)$



Ellipses Practice Quiz

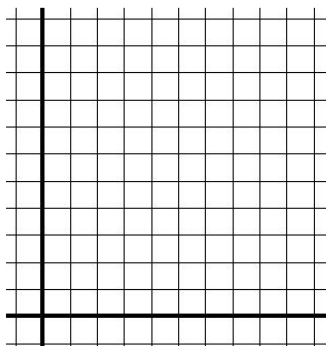
III. Graphing Ellipses

Graph the following ellipses; label the center, foci, and vertices. Find the *eccentricity*.

A) Center: (4, 5)

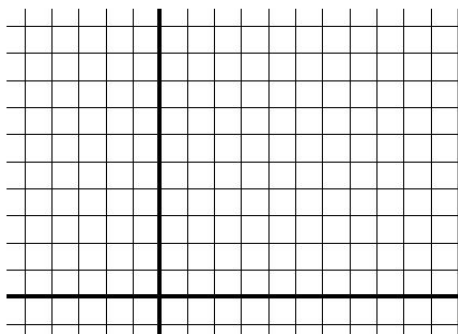
endpoint of major axis: (4, 8)

endpoint of minor axis: (6, 5)



B) Foci: (-3, 4) (9, 4)

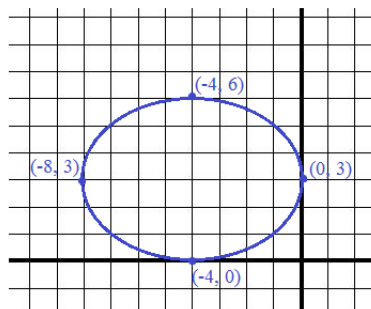
Length of major axis: 14



IV: Applications

A) Identify 3 other random points on this ellipse:

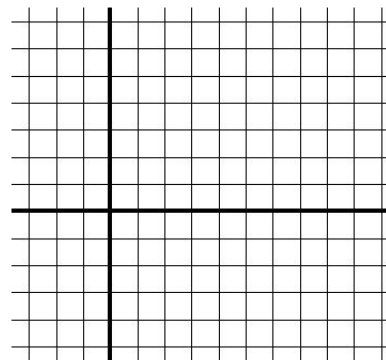
Then, verify your choices *algebraically*.



B) Given: ellipse $4x^2 - 24x + 9y^2 + 36y + 36 = 0$
line $2x + 3y = 6$

What are the points of intersection?

Solve algebraically and verify graphically.



C) The equation for Ellipse E is $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$

Express Ellipse E in *general form*.

Write the equation for the ellipse *reflected over the y-axis*. (Ellipse E')

Express E' in both *general form* and *standard form*

Ellipses Practice Quiz

SOLUTIONS

Standard form of an ellipse: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$
where (h, k) is the center

I. Parts of an Ellipse

Identify the center, vertices, co-vertices, foci, and 1 random point on each ellipse:

A) $\frac{(x+1)^2}{4} + \frac{(y+4)^2}{25} = 1$ ("vertical ellipse")

center: $(-1, -4)$

vertices: $(-1, 1)$ $(-1, -9)$

co-vertices: $(-3, -4)$ $(1, -4)$

foci: $(-1, -4 + \sqrt{21})$
 $(-1, -4 - \sqrt{21})$

random point: $(0, \frac{5\sqrt{3}-4}{2})$

if $x = 0$, then $\frac{(y+4)^2}{25} = \frac{3}{4}$

$b = 5$
vertex 5 above center
vertex 5 below center

$a = 2$
co-vertex 2 left center
co-vertex 2 right center

$c^2 = b^2 - a^2$
 $c = \pm \sqrt{21}$

approx: $(0, .33)$

B) $4x^2 + 9y^2 = 36$ $\frac{x^2}{9} + \frac{y^2}{4} = 1$

center: $(0, 0)$

vertices: $(3, 0)$ $(-3, 0)$

co-vertices: $(0, 2)$ $(0, -2)$

foci: $(-\sqrt{5}, 0)$ $(\sqrt{5}, 0)$

random point: $(2, \frac{2\sqrt{5}}{3})$

let $x = 2$, then $\frac{y^2}{4} = \frac{5}{9}$

("horizontal ellipse")

$c^2 = a^2 - b^2$
 $c = \pm \sqrt{5}$

approx: $(2, 1.49)$

C) $\frac{x^2}{5} + \frac{4(y-3)^2}{9} = 1$ $\frac{x^2}{5} + \frac{(y-3)^2}{9/4} = 1$

center: $(0, 3)$

vertices: $(-\sqrt{5}, 3)$ $(\sqrt{5}, 3)$

co-vertices: $(0, 9/2)$ $(0, 3/2)$

foci: $(-\frac{\sqrt{11}}{2}, 3)$ $(\frac{\sqrt{11}}{2}, 3)$

random point: $(5/3, 2)$

let $y = 2$, then $\frac{x^2}{5} = \frac{5}{9}$

("horizontal ellipse")

$a = \sqrt{5}$
 $b = 3/2$
 $c = \frac{\sqrt{11}}{2}$

D) $3x^2 + 4y^2 - 18x + 8y + 19 = 0$ (complete the square)

center: $(3, -1)$

vertices: $(5, -1)$ $(1, -1)$

co-vertices: $(3, -1 + \frac{\sqrt{3}}{2})$
 $(3, -1 - \frac{\sqrt{3}}{2})$

foci: $(4, -1)$ $(2, -1)$

random point: $(4, 1/2)$

$3x^2 - 18x + 4y^2 + 8y = -19$
 $3(x^2 - 6x + 9) + 4(y^2 + 2y + 1) = -19 + 27 + 4$

$3(x-3)^2 + 4(y+1)^2 = 12$

$\frac{(x-3)^2}{4} + \frac{(y+1)^2}{3} = 1$

$a = \frac{2}{\sqrt{3}}$
 $b = \sqrt{3}$

$c = \sqrt{4-3} = 1$

If $x = 4$, $\frac{(y+1)^2}{3} = \frac{3}{4}$ $y = 1/2$

E) $x^2 + 8x + 3y^2 - 12y = -1$

center: $(-4, 2)$

vertices: $(-4 + 3\sqrt{3}, 2)$
 $(-4 - 3\sqrt{3}, 2)$

co-vertices: $(-4, 5)$ $(-4, -1)$

foci: $(-4 + 3\sqrt{2}, 2)$
 $(-4 - 3\sqrt{2}, 2)$

random point: $(0, 3.91)$

If $x = 0$, then $\frac{(y-2)^2}{9} = \frac{11}{27}$

$y = 2 + \sqrt{\frac{11}{3}}$

(complete the square)

$x^2 + 8x + 16 + 3(y^2 - 4y + 4) = -1 + 16 + 12$

$(x+4)^2 + 3(y-2)^2 = 27$

$\frac{(x+4)^2}{27} + \frac{(y-2)^2}{9} = 1$

$a = 3$
 $b = \sqrt{27} = 3\sqrt{3}$

$c = \sqrt{18} = 3\sqrt{2}$

II. Equation of an Ellipse

Determine the equation of each ellipse from the given information.
(Express your answer in both standard form and general form)

A) Foci: $(2, 7 + \sqrt{5})$
 $(2, 7 - \sqrt{5})$

One Vertex: $(0, 7)$

midpoint of the foci is the center

center: $(2, 7)$

foci have the same x term, therefore
it is a "vertical ellipse"

If vertex is $(0, 7)$
and center is $(2, 7)$,
then other minor vertex
is $(4, 7)$...

$b = 2$

$c^2 = a^2 - b^2$

$5 = a^2 - (2)^2$

$a = 3$

$\frac{(x-2)^2}{4} + \frac{(y-7)^2}{9} = 1$

$(2, 10)$ and $(2, 4)$ are the
major vertices

$9(x-2)^2 + 4(y-7)^2 = 36$

$9x^2 + 4y^2 - 36x - 56y + 196 = 0$

B) Major Intercepts: $(0, 6)$ $(0, -6)$
Minor Intercepts: $(3, 0)$ $(-3, 0)$

midpoint of major intercept: $(0, 0)$
midpoint of minor intercept: $(0, 0)$
center is $(0, 0)$

length of a: $(0, 0)$ to $(0, 6) = 6$ ("vertical radius")
length of b: $(0, 0)$ to $(-3, 0) = 3$ ("horizontal radius")

$\frac{(x-0)^2}{9} + \frac{(y-0)^2}{36} = 1$

change to general form: (multiply by 36)

$4x^2 + y^2 = 36$

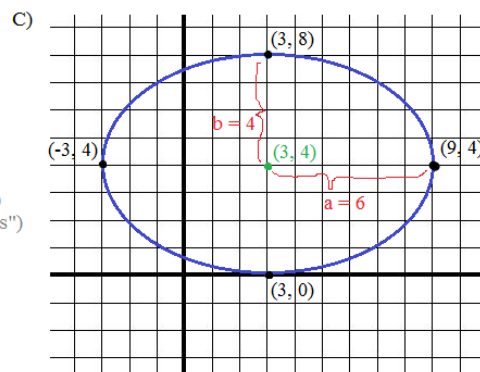
$4x^2 + y^2 - 36 = 0$

test intercepts to check
your equations!

General Form of an ellipse:

$Ax^2 + Cy^2 + Dx + Ey + F = 0$

$A \neq 0$ and A and C have same sign



$a = 6$ ("horizontal radius")
 $b = 4$ ("vertical radius")
center: $(3, 4)$

$\frac{(x-3)^2}{36} + \frac{(y-4)^2}{16} = 1$

(multiply by 144)

$4(x-3)^2 + 9(y-4)^2 = 144$

$4[x^2 - 6x + 9] + 9[y^2 - 8y + 16] = 144$

$4x^2 + 9y^2 - 24x - 72y + 36 = 0$

Ellipses Practice Quiz

a = "semi-major axis length"
b = "semi-minor axis length"

SOLUTIONS

$$\text{eccentricity: } e = \frac{c}{a}$$

$$0 < e < 1$$

$$\frac{(x-3)^2}{49} + \frac{(y-4)^2}{13} = 1$$

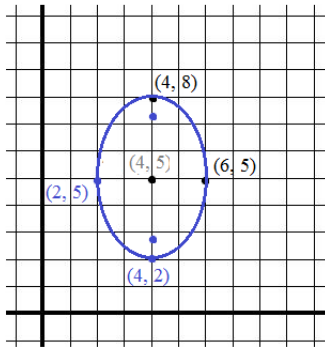
"horizontal ellipse"

III. Graphing Ellipses

Graph the following ellipses; label the center, foci, and vertices. Find the *eccentricity*.

A) Center: (4, 5)

endpoint of major axis: (4, 8)
endpoint of minor axis: (6, 5)



Center: (4, 5)
Major vertices: (4, 8) (4, 2)
a = 3
Minor vertices: (2, 5) (6, 5)
(co-vertices)
b = 2

$$c^2 = 3^2 - 2^2 = \frac{(x-4)^2}{4} + \frac{(y-5)^2}{9} = 1$$

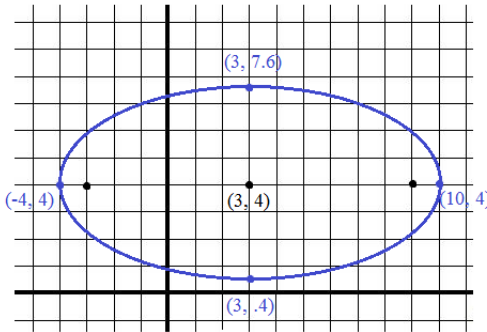
$$c = \sqrt{5}$$

$$e = \frac{\sqrt{5}}{3} \approx .745$$

foci: (4, 5 + $\sqrt{5}$) approx. (4, 7.23)
(4, 5 - $\sqrt{5}$) (4, 2.76)

B) Foci: (-3, 4) (9, 4)

Length of major axis: 14



Center: (3, 4)
(midpoint of foci)

a = 7
(length of 1/2 major axis)

c = 6
(distance from center to each focus)

$$e = \frac{6}{7} \approx .857$$

$$b = \sqrt{13}$$

$$c^2 = a^2 - b^2$$

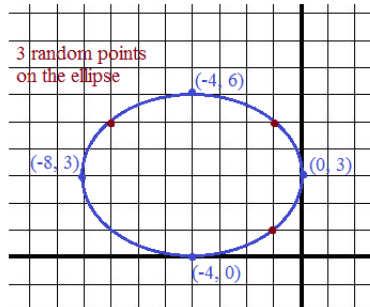
minor vertices:
(3, 4 ± $\sqrt{13}$)

note: .857 > .745 therefore, the ellipse is "more oval"...

IV. Applications

A) Identify 3 other random points on this ellipse:

Then, verify your choices *algebraically*.



(determine equation of ellipse)

center: (-4, 3)
a = 4
b = 3

$$\frac{(x+4)^2}{16} + \frac{(y-3)^2}{9} = 1$$

(change to general form)

$$9(x+4)^2 + 16(y-3)^2 = 144$$

$$9x^2 + 72x + 144 + 16y^2 - 96y + 144 = 144$$

$$9x^2 + 72x + 16y^2 - 96y + 144 = 0$$

(substitution to find points!)

Let x = -1

$$9(-1)^2 + 72(-1) + 16y^2 - 96y + 144 = 0$$

$$16y^2 - 96y + 81 = 0$$

(factor/quadratic formula)

$$y \approx \frac{96 \pm 63.5}{32} \text{ approximately: } 1.02 \text{ and } 4.98$$

Let x = -7 (substitute)

$$\frac{(-7+4)^2}{16} + \frac{(y-3)^2}{9} = 1$$

$$\frac{(y-3)^2}{9} = 1 - \frac{9}{16} \quad y - 3 = \pm \frac{\sqrt{63}}{4}$$

(note: there is axis symmetry)

(-1, 1.02) (-1, 4.98)

(-7, 3 + $\frac{\sqrt{63}}{4}$)

B) Given: ellipse $4x^2 - 24x + 9y^2 + 36y + 36 = 0$
line $2x + 3y = 6$

What are the points of intersection?

Solve algebraically and verify graphically.

$$x = \frac{6-3y}{2} \text{ then, substitute into ellipse}$$

$$4\left(\frac{6-3y}{2}\right)^2 - 24\left(\frac{6-3y}{2}\right) + 9y^2 + 36y + 36 = 0$$

$$(6-3y)^2 - 12(6-3y) + 9y^2 + 36y + 36 = 0$$

$$36 - 36y + 9y^2 - 72 + 36y + 9y^2 + 36y + 36 = 0$$

$$18y^2 + 36y = 0$$

$$18y(y+2) = 0$$

C) The equation for Ellipse E is $\frac{(x-2)^2}{4} + \frac{(y+3)^2}{9} = 1$

Express Ellipse E in *general form*.

Write the equation for the ellipse *reflected over the y-axis*. (Ellipse E')

Express E' in both *general form* and *standard form*

ellipse in standard form:

$$4(x^2 - 6x + 9) + 9(y^2 + 4y + 4) = -36 + 72$$

$$4(x-3)^2 + 9(y+2)^2 = 36$$

$$\frac{(x-3)^2}{9} + \frac{(y+2)^2}{4} = 1$$

y = 0
then,
x = 3

(3, 0)

y = -2
then,
x = 6

(6, -2)

When reflected, the ellipse maintains the shape -- but, the center changes....

(center changes from (2, -3) to (-2, -3))

$$E': \frac{(x+2)^2}{4} + \frac{(y+3)^2}{9} = 1$$

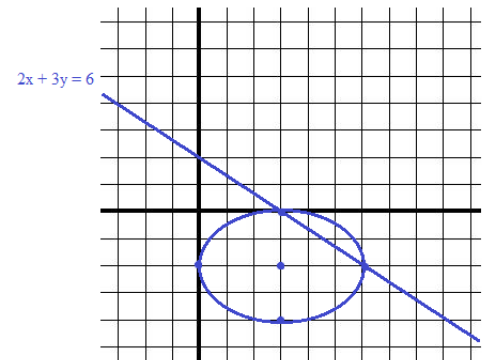
$$E': 9(x+2)^2 + 4(y+3)^2 = 36$$

$$9x^2 + 4y^2 + 36x + 24y + 36 = 0$$

$$E: 9(x-2)^2 + 4(y+3)^2 = 36$$

$$9x^2 + 4y^2 - 36x + 24y + 36 = 0$$

(Note: when reflected over the y-axis, the coefficient of x ("D term") changes signs!)



HYPERBOLAS

Math Olympics
Day 9

"Stay calm! You can't lose your *focus*!"

"I know! I know! And stay *centered*!"

Two *Hyper Bolas* before the Graphing Competition

Official Corporate Sponsors

Power Ade
A math boost to the n^{th} power

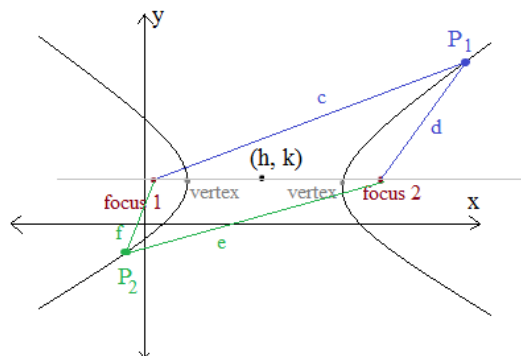
Remember to drink and calculate responsibly

official airline of the math olympics.

Hyperbola

What is it? A 'reversed', 'inside-out' ellipse.

A set of points that have the same 'difference between the distances to 2 given points (foci)'



Horizontal Hyperbola

Comments:

(h, k) is the center

The **foci** and **vertices** are on the same horizontal axis ($y = k$)

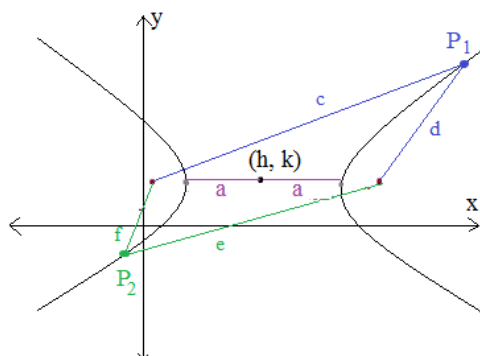
$$e - f = c - d$$

Comments:

a is the distance from the center to each vertex.

Vertices are $(h - a, k)$ and $(h + a, k)$

$$c - d = 2a$$



General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$

where $A \neq C$

and

A and C have opposite signs
(neither can be 0)

Standard Form:

$$\frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1$$

where (h, k) is the center

Slopes of the Asymptotes are $\pm \frac{b}{a}$

Hyperbolas: Finding the Focus, Graphing, and The "Box"

Notes on Conics

Example 1: $x^2 - 4y^2 - 8x + 16y - 36 = 0$

Complete the square to transform General Form into Standard Form.

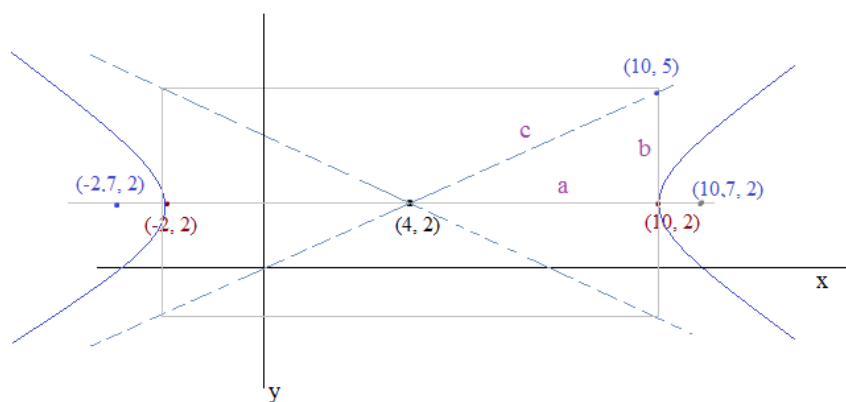
$$x^2 - 8x - 4y^2 + 16y = 36$$

$$x^2 - 8x - 4(y^2 - 4y) = 36$$

$$x^2 - 8x + 16 - 4(y^2 - 4y + 4) = 36 + 16 - 4(4)$$

$$(x - 4)^2 - 4(y - 2)^2 = 36$$

$$\frac{(x - 4)^2}{36} - \frac{(y - 2)^2}{9} = 1$$



Comments:

To check work, plug the vertices into the original equation:

$$\begin{aligned} (-2, 2) \quad & (-2)^2 - 4(2)^2 - 8(-2) + 16(2) - 36 = 0 \\ & 4 - 16 + 16 + 32 - 36 = 0 \quad \checkmark \end{aligned}$$

$$\begin{aligned} (10, 2) \quad & (10)^2 - 4(2)^2 - 8(10) + 16(2) - 36 = 0 \\ & 100 - 16 - 80 + 32 - 36 = 0 \quad \checkmark \end{aligned}$$

$$h = 4$$

$$k = 2$$

$$a = 6$$

$$b = 3$$

$36 > -9$, so it's a "horizontal hyperbola"

Center: (4, 2)

Vertices: (10, 2) and (-2, 2)

$$\begin{aligned} \text{Focus: } c^2 &= a^2 + b^2 \\ &= 36 + 9 \\ c &= \pm 3\sqrt{5} \end{aligned}$$

$$(4 + 3\sqrt{5}, 2) \text{ and } (4 - 3\sqrt{5}, 2)$$

$$(10.7, 2) \quad (-2.7, 2)$$

Asymptotes are lines through the center with slopes:

$$\frac{3}{6} \text{ and } -\frac{3}{6}$$

$$\frac{b}{a} \quad \frac{-b}{a}$$

$$y = \frac{1}{2}x \text{ and } y = -\frac{1}{2}x + 4$$

(asymptotes)

ellipse -- sum of the distance to each focus
hyperbola -- difference of the distance to each focus

Hyperbola:

$$\frac{(x - 4)^2}{36} - \frac{(y - 2)^2}{9} = 1$$

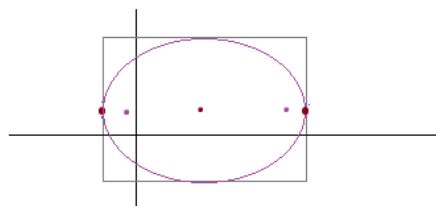
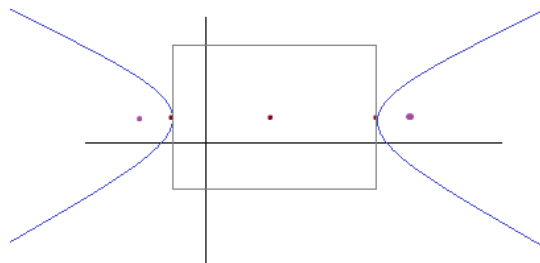
$$c^2 = a^2 + b^2$$

(Comparing hyperbola & ellipse with the same a, b, and center)

Ellipse:

$$\frac{(x - 4)^2}{36} + \frac{(y - 2)^2}{9} = 1$$

$$c^2 = a^2 - b^2$$



Hyperbolas: Vertical Hyperbola

Notes on Conics

Example 2: $9x^2 - 16y^2 + 90x + 32y + 353 = 0$

Complete the square to put into Standard Form.

$$9x^2 + 90x - 16y^2 + 32y = -353$$

$$9(x^2 + 10x) - 16(y^2 - 2y) = -353$$

$$9(x^2 + 10x + 25) - 16(y^2 - 2y + 1) = -353 + 9(25) - 16(1)$$

$$9(x + 5)^2 - 16(y - 1)^2 = -144$$

$$-\frac{(x + 5)^2}{16} + \frac{(y - 1)^2}{9} = 1$$

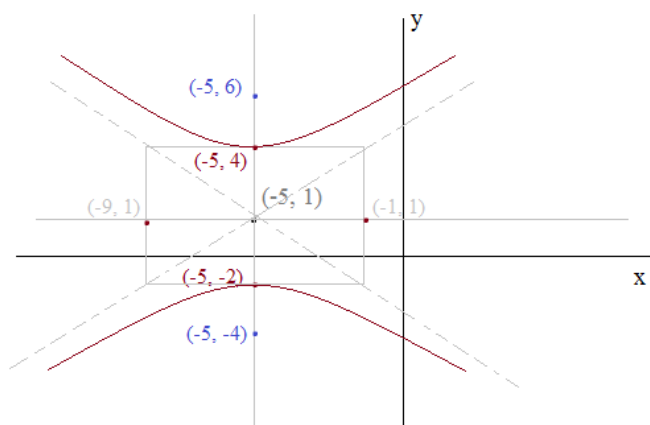
$$\frac{(y - 1)^2}{9} - \frac{(x + 5)^2}{16} = 1$$

Since the x term is negative and the y term is positive, it is a vertical hyperbola.

Center: $(-5, 1)$ $a = 4$
 $b = 3$

$$c = \sqrt{16 + 9} = \pm 5$$

Asymptote slopes: $\pm \frac{3}{4}$



Vertices: $(-5, 1 - 3)$
 $(-5, 1 + 3)$

Foci: $(-5, 1 - 5)$
 $(-5, 1 + 5)$

Comment: To verify the hyperbola is vertical, test the potential vertices.

$$9x^2 - 16y^2 + 90x + 32y + 353 = 0$$

$(-5, 4)$ $9(-5)^2 - 16(4)^2 + 90(-5) + 32(4) + 353 = 0$
 $225 - 256 - 450 + 128 + 353 = 0$
 $0 = 0$ ✓

$(-5, -2)$ $9(-5)^2 - 16(-2)^2 + 90(-5) + 32(-2) + 353 = 0$
 $225 - 64 - 450 - 64 + 353 = 0$
 $0 = 0$ ✓

$(-9, 1)$ $9(-9)^2 - 16(1)^2 + 90(-9) + 32(1) + 353 = 0$
 $729 - 16 - 810 + 32 + 353 = 0$
 $288 \neq 0$ ✗

$(-1, 1)$ $9(-1)^2 - 16(1)^2 + 90(-1) + 32(1) + 353 = 0$
 $9 - 16 - 90 + 32 + 353 = 0$
 $288 \neq 0$ ✗

These points are NOT on the hyperbola.
 Therefore, the hyperbola must be vertical.

Hyperbola and Ellipse

Conic System: Hyperbola and Ellipse

Solve and Graph the following system:

$$7x^2 - 9y^2 = 31$$

$$5x^2 + 9y^2 = 161$$

Algebraically:

$$7x^2 - 9y^2 = 31$$

$$5x^2 + 9y^2 = 161$$

(use combination method)

$$12x^2 = 192$$

$$x^2 = 16$$

$$x = \pm 4$$

(plug x into 2nd equation to find y)

$$5(4)^2 + 9y^2 = 161$$

$$9y^2 = 81$$

$$y = \pm 3$$

$$5(-4)^2 + 9y^2 = 161$$

$$9y^2 = 81$$

$$y = \pm 3$$

Solutions: (4, 3) (4, -3) (-4, 3) (-4, -3)

Graphically:

$$5x^2 + 9y^2 = 161$$

$$\frac{5}{161}x^2 + \frac{9}{161}y^2 = 1$$

$$\frac{x^2}{\left(\frac{161}{5}\right)} + \frac{y^2}{\left(\frac{161}{9}\right)} = 1$$

Horizontal Ellipse

$$a = \sqrt{\left(\frac{161}{5}\right)} \quad 5.67$$

$$b = \sqrt{\left(\frac{161}{9}\right)} \quad 4.23$$

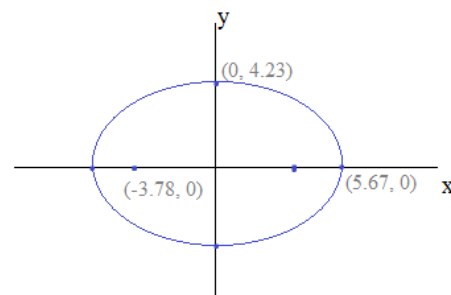
$$c = \pm \sqrt{\frac{644}{45}} = \pm \frac{2\sqrt{\frac{161}{5}}}{3} \quad 3.78$$

center: (0, 0)

Foci: (-3.78, 0) and (3.78, 0)

Major Vertices: (-5.67, 0) and (5.67, 0)

Minor Vertices: (0, -4.23) and (0, 4.23)



Horizontal Hyperbola

$$7x^2 - 9y^2 = 31$$

$$\frac{x^2}{\left(\frac{31}{7}\right)} - \frac{y^2}{\left(\frac{31}{9}\right)} = 1$$

$$a = \sqrt{\left(\frac{31}{7}\right)} \quad 2.10$$

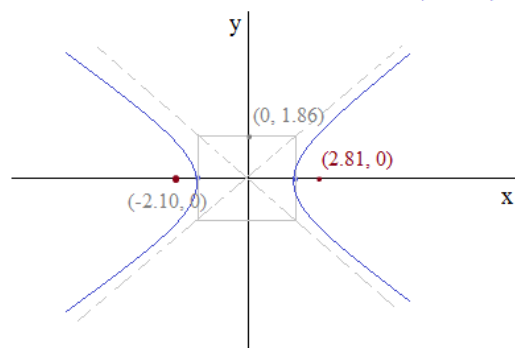
$$b = \sqrt{\left(\frac{31}{9}\right)} \quad 1.86$$

$$c = \pm \sqrt{\frac{496}{63}} = \pm \frac{4\sqrt{\frac{31}{7}}}{3} \quad 2.81$$

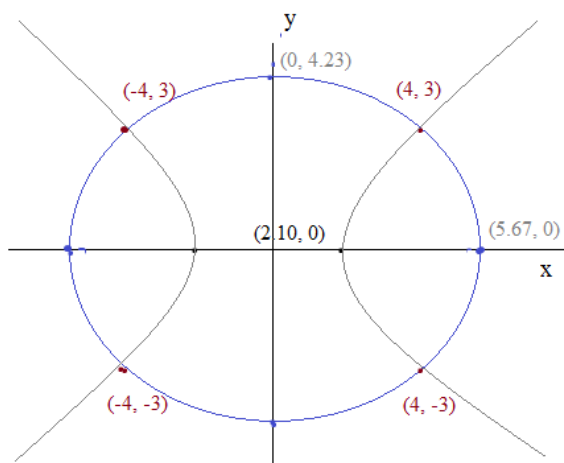
center: (0, 0)

Foci: (2.81, 0) and (-2.81, 0)

Vertices: (2.10, 0) and (-2.10, 0)



Conic System: Hyperbola and Ellipse Graph



I. Parts of a Hyperbola

Identify the center, vertices, foci, and asymptotes of each hyperbola:

A) $\frac{(x-2)^2}{25} - \frac{(y+1)^2}{9} = 1$

center: _____

vertices: _____

foci: _____

asymptotes: _____

B) $4x^2 - 9y^2 = 144$

center: _____

vertices: _____

foci: _____

asymptotes: _____

C) $\frac{-x^2}{9} + \frac{(y+4)^2}{49} = 1$

center: _____

vertices: _____

foci: _____

asymptotes: _____

D) $4x^2 - 3y^2 + 40x + 18y + 61 = 0$

center: _____

vertices: _____

foci: _____

asymptotes: _____

E) $9x^2 - y^2 + 6y = 18$

center: _____

vertices: _____

foci: _____

asymptotes: _____

II. Equation of a Hyperbola

Determine the equation of each hyperbola from the given information.
(Express your answer in both *standard* form and *general* form)

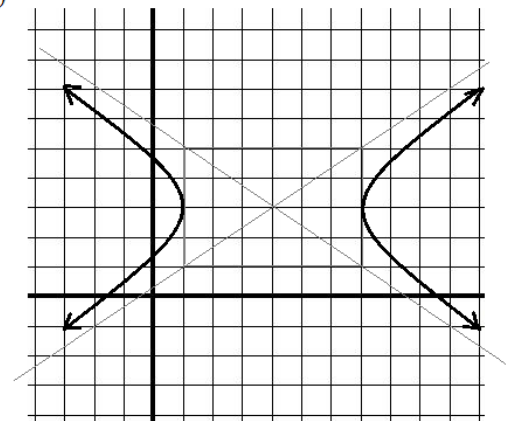
A) Vertices: (3, 0) (7, 0)

Vertical distance to the "box":
("semi-minor axis length")
4 units

B) Foci: (-2, -2) (-2, 8)

Vertices: (-2, 0) (-2, 6)

C)

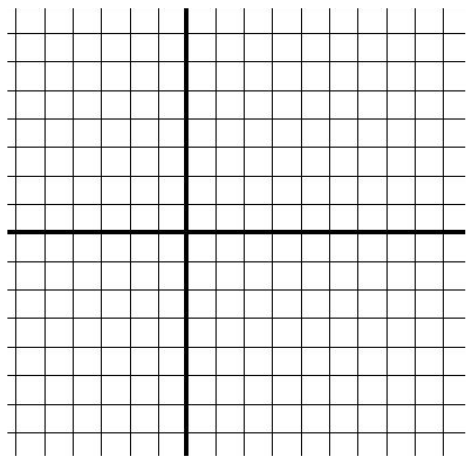


Hyperbolas Practice Quiz

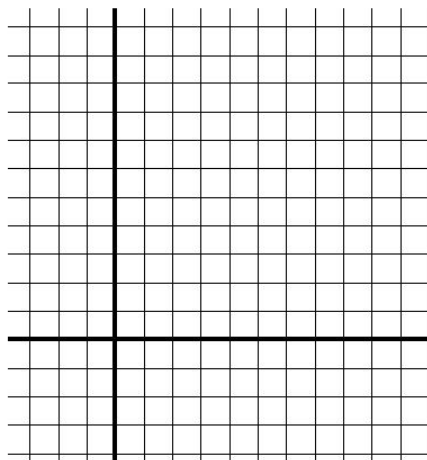
III. Graphing Hyperbolas

Graph the following hyperbolas; label the center, vertices, foci, and asymptotes.

A) $25x^2 - y^2 - 100x + 75 = 0$



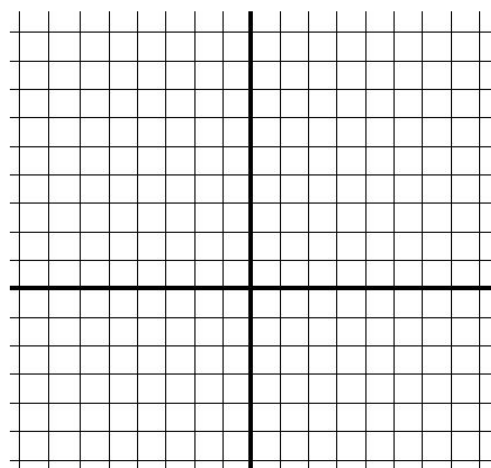
B) $10(y - 3)^2 - 4(x - 5)^2 = 40$



IV: Applications

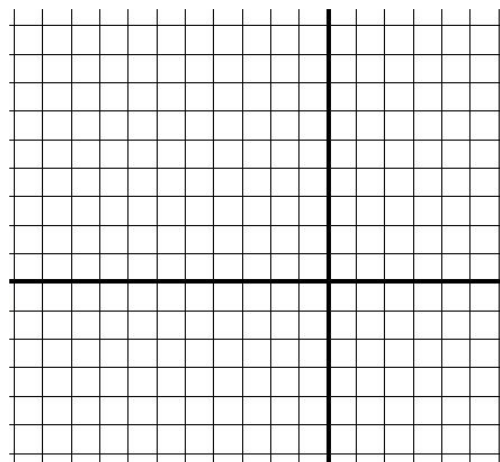
A) Given: hyperbola $4x^2 - y^2 = 4$
line $x + 3y = 1$

What are the points of intersection?
Solve algebraically and verify graphically.



B) Find the x and y-intercepts of the hyperbola.
Graph the hyperbola, labeling the intercepts.

$x^2 - y^2 + 4x + 8y = 13$



Hyperbolas Practice Quiz

SOLUTIONS

Standard Form: $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
 ("horizontal" hyperbola;
 horizontal line segment
 would connect vertices)

I. Parts of a Hyperbola

Identify the center, vertices, foci, and asymptotes of each hyperbola:

A) $\frac{(x-2)^2}{25} - \frac{(y+1)^2}{9} = 1$
 center: $(h, k) = (2, -1)$
 vertices: $(7, -1)$ $(-3, -1)$
 foci: $(2 + \sqrt{34}, -1)$ $(2 - \sqrt{34}, -1)$
 asymptotes: $y + 1 = \frac{3}{5}(x - 2)$
 $y + 1 = -\frac{3}{5}(x - 2)$
 slope of asymptotes: $\frac{b}{a} = \frac{3}{5}$ $-\frac{b}{a} = -\frac{3}{5}$
 both asymptotes go through the center $(2, -1)$

B) $4x^2 - 9y^2 = 144$ (rewrite)
 $\frac{x^2}{36} - \frac{y^2}{16} = 1$
 center: $(0, 0)$
 vertices: $(6, 0)$ $(-6, 0)$
 foci: $(2\sqrt{13}, 0)$ $(-2\sqrt{13}, 0)$
 asymptotes: $y = (2/3)x$
 $y = (-2/3)x$
 slope: $4/6$ and $-4/6$
 asymptotes go through center (origin)

C) $\frac{-x^2}{9} + \frac{(y+4)^2}{49} = 1$ (rewrite)
 $\frac{(y+4)^2}{49} - \frac{x^2}{9} = 1$
 center: $(0, -4)$ "vertical" hyperbola
 vertices: $(0, -11)$ $(0, 3)$
 foci: $(0, -4 + \sqrt{58})$ $(0, -4 - \sqrt{58})$
 asymptotes: $y = 7x/3 - 4$
 $y = -7x/3 - 4$
 slope: b/a and $-b/a$
 $7/3$ and $-7/3$
 asymptotes through center $(0, -4)$

D) $4x^2 - 3y^2 + 40x + 18y + 61 = 0$
 (complete the square to change to standard form)
 $4x^2 + 40x - 3y^2 + 18y = -61$
 $4(x^2 + 10x + 25) - 3(y^2 - 6y + 9) = -61 + 100 - 27$
 $4(x+5)^2 - 3(y-3)^2 = 12$
 $\frac{(x+5)^2}{3} - \frac{(y-3)^2}{4} = 1$
 center: $(-5, 3)$
 vertices: $(-5 + \sqrt{3}, 3)$ $(-5 - \sqrt{3}, 3)$
 foci: $(-5 + \sqrt{7}, 3)$ $(-5 - \sqrt{7}, 3)$
 asymptotes: $y - 3 = \frac{2}{\sqrt{3}}(x + 5)$
 $y - 3 = -\frac{2}{\sqrt{3}}(x + 5)$
 slope of asymptotes: $\pm \frac{2}{\sqrt{3}}$

E) $9x^2 - y^2 + 6y = 18$
 $9x^2 - (y^2 - 6y + 9) = 18 - 9$
 $9x^2 - (y-3)^2 = 9$
 $x^2 - \frac{(y-3)^2}{9} = 1$
 center: $(0, 3)$
 vertices: $(-1, 3)$ $(1, 3)$
 foci: $(-\sqrt{10}, 3)$ $(\sqrt{10}, 3)$
 asymptotes: $y = 3x + 3$
 $y = -3x + 3$
 $h = 0$
 $k = 3$
 $a = 1$
 $b = 3$
 $c = \sqrt{10}$

II. Equation of a Hyperbola

Determine the equation of each hyperbola from the given information.
 (Express your answer in both *standard form* and *general form*)

A) Vertices: $(3, 0)$ $(7, 0)$

Vertical distance to the "box":
 ("semi-minor axis length")

4 units

Since the vertices are $(3, 0)$ and $(7, 0)$,
 the center is the midpoint: $(5, 0)$.

The "semi-major" axis is 2 ("a" = 2)

Since the "semi-minor axis length" is 4, "b" = 4

Standard form: $\frac{(x-5)^2}{4} - \frac{(y-0)^2}{16} = 1$

(multiply and change to general form)

$$4(x-5)^2 - 1(y-0)^2 = 16$$

$$4x^2 - 40x + 100 - y^2 = 16$$

$$4x^2 - y^2 - 40x + 84 = 0$$

(general form)

B) Foci: $(-2, -2)$ $(-2, 8)$

Vertices: $(-2, 0)$ $(-2, 6)$

center: $(-2, 3)$

since vertices are $(-2, 0)$ and $(-2, 6)$,
 the "semi-major" axis is 3 ("b" = 3)

("vertical hyperbola")

focal length: distance from center
 to focus = 5 ("c" = 5)

since $a^2 + b^2 = c^2$, "a" = 4

Standard Form: $\frac{(y-3)^2}{9} - \frac{(x+2)^2}{16} = 1$

(multiply by 144 to change to general form)

$$16(y-3)^2 - 9(x+2)^2 = 144$$

$$16y^2 - 96y + 144 - 9x^2 - 36x - 36 = 144$$

$$-9x^2 + 16y^2 - 36x - 96y - 36 = 0$$

C)

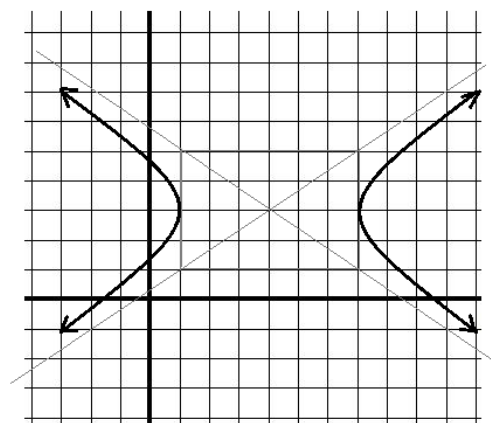
$$\frac{(x-4)^2}{9} - \frac{(y-3)^2}{4} = 1$$

center: $(4, 3)$

vertices: $(1, 3)$ $(7, 3)$

semi-major axis (a): 3 units

semi-minor axis (b): 2 units



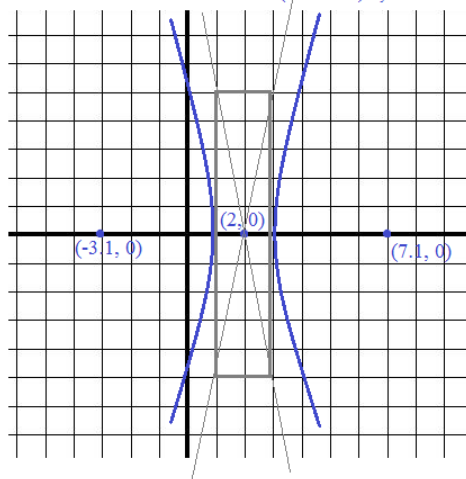
$$4(x-4)^2 - 9(y-3)^2 = 36$$

$$4x^2 - 9y^2 - 32x + 54y - 53 = 0$$

III. Graphing Hyperbolas

Graph the following hyperbolas; label the center, vertices, foci, and asymptotes.

A) $25x^2 - y^2 - 100x + 75 = 0$ $25(x^2 - 4x) - y^2 = -75$
 $25(x^2 - 4x + 4) - y^2 = -75 + 100$



$$25(x-2)^2 - y^2 = 25$$

$$(x-2)^2 - \frac{y^2}{25} = 1$$

center: (2, 0)

$$a = 1$$

$$b = 5$$

$$c = \sqrt{26}$$

asymptotes:

$$y = 5x - 10$$

$$y = -5x + 10$$

vertices: (1, 0) (3, 0)

foci: (2 + $\sqrt{26}$, 0)(2 - $\sqrt{26}$, 0)

B) $10(y-3)^2 - 4(x-5)^2 = 40$

$$\frac{(y-3)^2}{4} - \frac{(x-5)^2}{10} = 1$$

("vertical hyperbola")

center: (5, 3)

$$a = \sqrt{10}$$

$$b = 2$$

$$c = \sqrt{14}$$

vertices:

center + b: (5, 5)

center - b: (5, 1)

foci:

center + c: (5, 3 + $\sqrt{14}$)center - c: (5, 3 - $\sqrt{14}$)

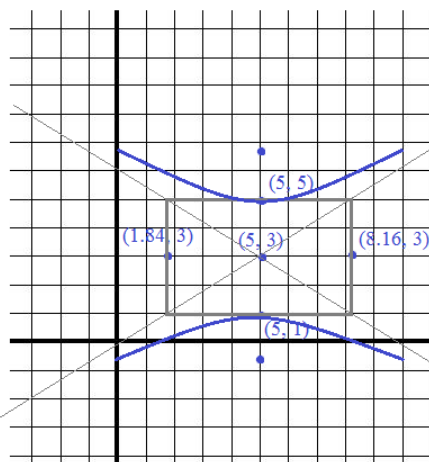
asymptotes:

$$\text{slope: } + \frac{2}{\sqrt{10}}$$

$$- \frac{2}{\sqrt{10}}$$

$$y - 3 = \frac{2}{\sqrt{10}}(x - 5)$$

$$y - 3 = -\frac{2}{\sqrt{10}}(x - 5)$$



IV. Applications

A) Given: hyperbola $4x^2 - y^2 = 4$
 line $x + 3y = 1$

What are the points of intersection?
Solve algebraically and verify graphically.

Using substitution, $4x^2 - y^2 = 4$

$$4(1 - 3y)^2 - y^2 = 4$$

$$4 - 24y + 36y^2 - y^2 = 4$$

$$35y^2 - 24y = 0$$

$$y(35y - 24) = 0$$

$$y = 0 \text{ and } 24/35$$

If $y = 0$, then

$$x + 3(0) = 1$$

$$x = 1$$

If $y = 24/35$, then

$$x + 3(24/35) = 1$$

$$x = -37/35$$

$$(1, 0) \quad \left(-\frac{37}{35}, \frac{24}{35}\right)$$

$$x^2 - \frac{y^2}{4} = 1$$

center: (0, 0)

$$a = 1$$

$$b = 2$$

vertices: (-1, 0) (1, 0)

asymptotes:

$$y = 2x \quad y = -2x$$

substitute to check answers:

$$4(-37/35)^2 - (24/35)^2 = 4$$

$$4.47 - .47 = 4 \quad \checkmark$$

$$4(1)^2 - (0)^2 = 4$$

$$4 = 4 \quad \checkmark$$

B) Find the x and y-intercepts of the hyperbola.

Graph the hyperbola, labeling the intercepts.

$$x^2 - y^2 + 4x + 8y = 13$$

The coordinates of the x-intercepts: (x, 0)

$$x^2 - 0 + 4x + 8(0) = 13$$

$$x^2 + 4x - 13 = 0 \quad (\text{quadratic formula})$$

$$x = -2 \pm \sqrt{17}$$

$$\text{approximately } (-6.1, 0) \quad (2.1, 0)$$

The coordinates of the y-intercepts: (0, y)

$$0 - y^2 + 4(0) + 8y = 13$$

$$y^2 - 8y + 13 = 0$$

$$\text{approximately } (0, 5.7) \quad (0, 2.3)$$

$$y = 4 \pm \sqrt{3}$$

put hyperbola into standard form:

$$(x^2 + 4x + 4) - (y^2 - 8y + 16) = 13 + 4 - 16$$

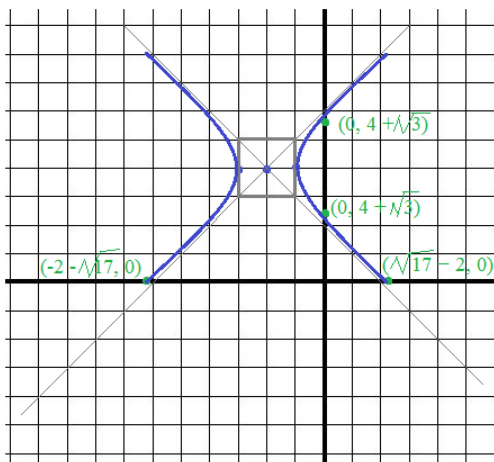
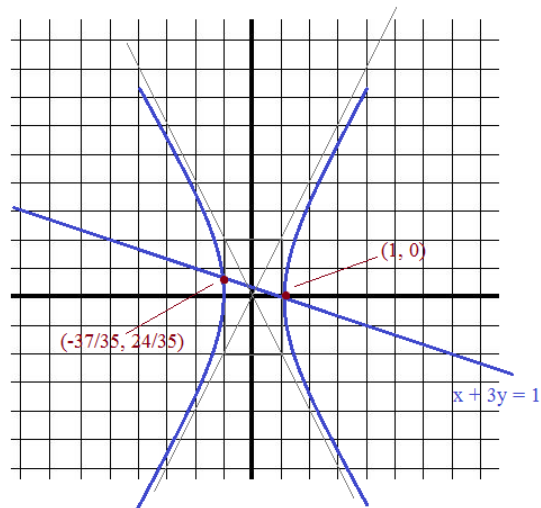
$$(x+2)^2 - (y-4)^2 = 1$$

center: (-2, 4)

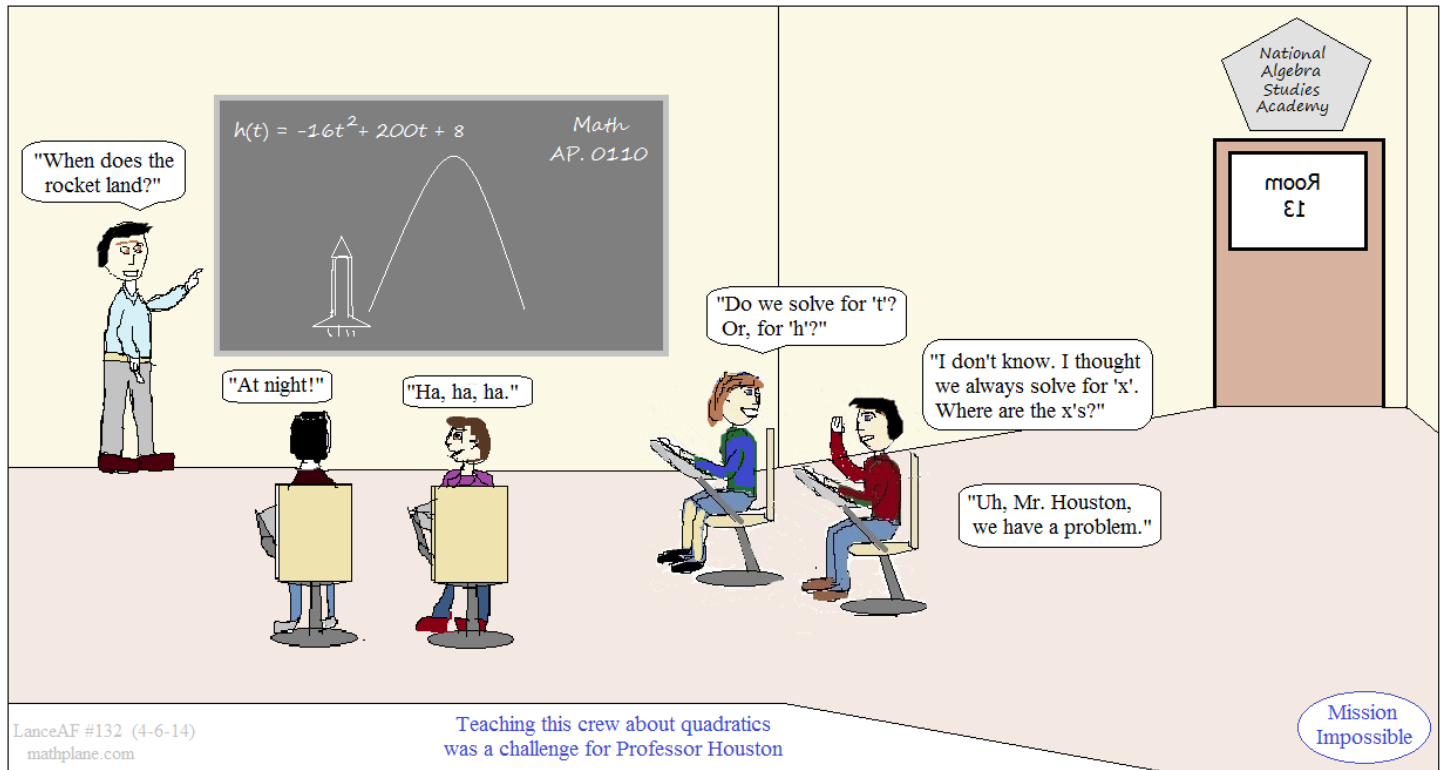
$$a = 1$$

$$b = 1$$

vertices: (-1, 4) (-3, 4)

asymptotes: $y = x + 6$
 $y = -x + 2$ 

Parabola



Parabola

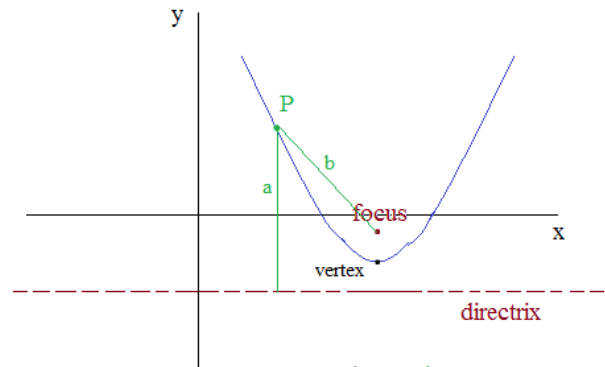
Notes on Conics

What is it? A set of (coplanar) points that have the same distance to a given point (focus) and to a line (directrix) that does not go through the focus.

Comments:

- It is 'U-shaped'
- A parabola has a quadratic equation.
- A parabola is '1/2 a hyperbola'.
- vertex is halfway between focus and directrix

General Form: $Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$
where A or C is 0



example: $a = b$
distance P to focus =
distance P to directrix

Standard Form: $y = a(x - h)^2 + k$ (vertical parabola: there is no y^2 term)

Vertex: (h, k)
if $a > 0$, parabola opens up
if $a < 0$, parabola opens down

Focus: $(h, k + p)$
where $p = \frac{1}{4a}$

Directrix: $y = k - p$

Example 1: $y = x^2 - 10x + 21$

There is no y^2 term, so we know it's a (vertical) parabola.

General Form: $x^2 - 10x - y + 21 = 0$

Standard Form (Also, called 'Vertex Form')
(Use 'completing the square' to transform.)

$$\begin{aligned} x^2 - 10x - y + 21 &= 0 \\ x^2 - 10x + 25 - y + 21 &= 0 + 25 \\ (x - 5)(x - 5) - y &= 4 \\ (x - 5)^2 &= y + 4 \\ y &= (x - 5)^2 - 4 \end{aligned}$$

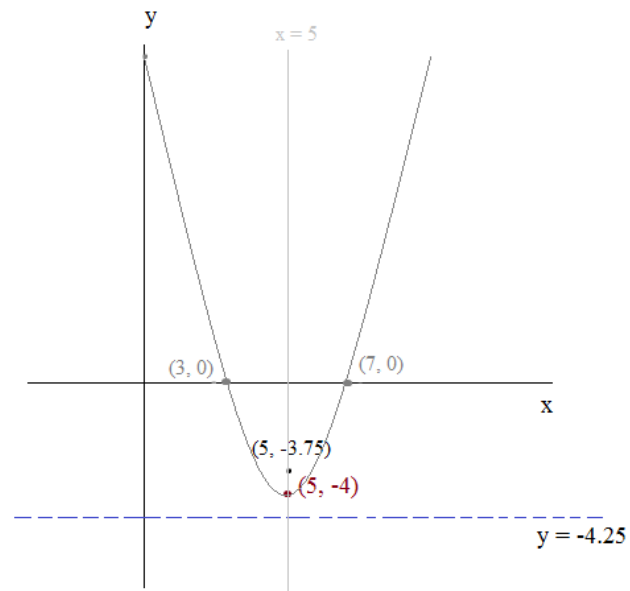
Vertex: $(5, -4)$

$a = 1$ (parabola opens up)

$p = \frac{1}{4} = .25$

Focus: $(5, -3.75)$

Directrix: $y = -4.25$



x-intercept: $(0, 21)$

y-intercepts: $(3, 0)$ $(7, 0)$

axis of symmetry: $x = 5$

Example 2: $y^2 - 5x - 10y - 15 = 0$

Since there is no x^2 term, this is a (horizontal) parabola.

Complete the square to transform into Standard Form.

$$y^2 - 10y - 5x - 15 = 0$$

$$y^2 - 10y + 25 - 5x - 15 = 0 + 25$$

$$(y - 5)(y - 5) - 5x = 40$$

$$(y - 5)^2 = 40 + 5x$$

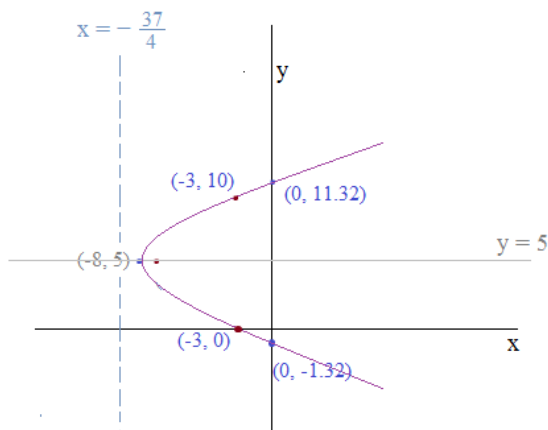
$$5x = (y - 5)^2 - 40$$

$$x = \frac{1}{5}(y - 5)^2 - 8$$

Comments: To check work and verify the sketch, consider the axis of symmetry, vertex, and intercepts.

vertex $(-8, 5)$ $y^2 - 5x - 10y - 15 = 0$
 $(5)^2 - 5(-8) - 10(5) - 15 = 0$
 $25 + 40 - 50 - 15 = 0$
 $0 = 0$ ✓

axis of symmetry: $y = 5$
 (horizontal line
 through the vertex)



$$a = \frac{1}{5} \quad a > 0, \text{ so the parabola opens to the right.}$$

$$h = -8$$

$$k = 5$$

vertex: $(-8, 5)$

$$p = \frac{1}{4a} = \frac{5}{4}$$

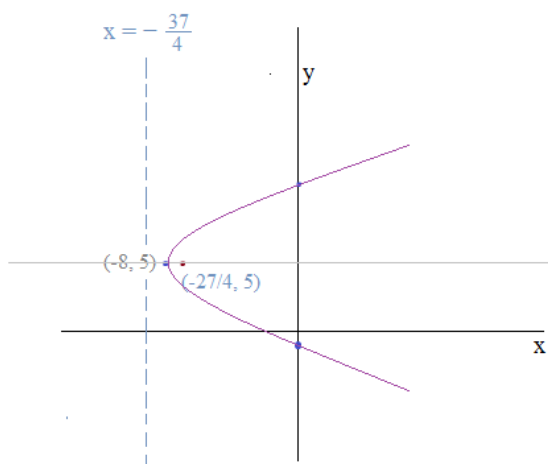
$$\text{directrix: } x = \frac{-37}{4}$$

$$\text{focus: } \left(\frac{-27}{4}, 5 \right)$$

$h \quad p$

$$-8 - 5/4 = -37/4$$

$$-8 + 5/4 = -27/4$$



y-intercepts: $y^2 - 5x - 10y - 15 = 0$

$$y^2 - 5(0) - 10y - 15 = 0$$

$$y^2 - 10y - 15 = 0$$

using quadratic formula,

y is approximately 11.32 or -1.32

$$(0, 11.32) \text{ and } (0, -1.32)$$

x-intercept: $y^2 - 5x - 10y - 15 = 0$

$$(0)^2 - 5x - 10(0) - 15 = 0$$

$$x = -3$$

$$(-3, 0)$$

Since $(-3, 0)$ is a point on the parabola and the axis of symmetry is $y = 5$, then $(-3, 10)$ is also a point on the parabola.

(i.e. same distance above and below the axis of symmetry)

Finding the "p" value in a parabola

If you look at various math resources, you may see different approaches to finding the "p" value.
 (***)The formulas will depend on the position of the terms in Standard form.)

mathplane.com

Option 1: The "a" value is with the "quadratic term"

$$a = \frac{1}{4p} \quad p = \frac{1}{4a} \quad y = \boxed{a(x-h)^2} + k$$

$$\text{Standard form: } y = a(x-h)^2 + k$$

where (h, k) is vertex

("Vertical Parabola")

$$p = \frac{1}{4a} \quad \text{or} \quad a = \frac{1}{4p}$$

$$\text{Also, } y - k = a(x-h)^2$$

$$\text{Standard form: } x = a(y-k)^2 + h$$

where (h, k) is vertex

("Horizontal Parabola")

$$p = \frac{1}{4a} \quad \text{or} \quad a = \frac{1}{4p}$$

$$\text{Also, } x - h = a(y-k)^2$$

Option 2: The "a" value is with the "linear term"

$$a = 4p \quad \boxed{a(y-k)} = (x-h)^2$$

$$\text{Standard form: } a(y-k) = (x-h)^2$$

where (h, k) is the vertex

("Vertical Parabola")

$$a = 4p$$

$$\text{Standard form: } a(x-h) = (y-k)^2$$

where (h, k) is the vertex

("Horizontal Parabola")

$$a = 4p$$

Example: For the following parabola, find the distance between the vertex and focus.

$$x^2 + 4x - 8y + 36 = 0$$

This is a "vertical parabola" because there is an x^2 term, but there is no y^2 term...

First, let's convert the equation from General form into Standard form.
 (Complete the square)

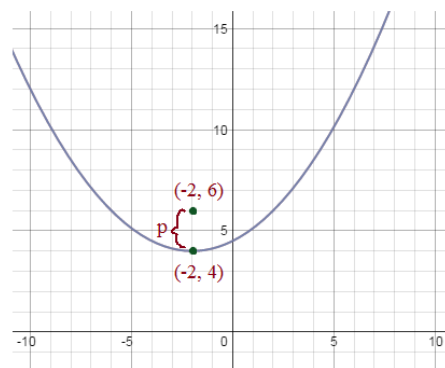
$$x^2 + 4x = 8y - 36$$

$$x^2 + 4x + 4 = 8y - 36 + 4$$

$$(x+2)^2 = 8y - 32$$

$$(x+2)^2 = 8(y-4)$$

The vertex (h, k) is (-2, 4)



Then, using the above formulas, find the "p" value...

$$(x+2)^2 = 8(y-4)$$

OR

$$\frac{1}{8}(x+2)^2 = (y-4) \quad \text{or} \quad y = \frac{1}{8}(x+2)^2 + 4$$

(the "a" value is grouped with the linear term)

(the "a" value is grouped with the quadratic term)

$$a = 4p$$

$$8 = 4p$$

$$p = 2$$

$$a = \frac{1}{4p}$$

$$\frac{1}{8} = \frac{1}{4p}$$

$$p = 2$$

$$p = \frac{1}{4a}$$

$$p = \frac{1}{4(1/8)}$$

$$p = 2$$

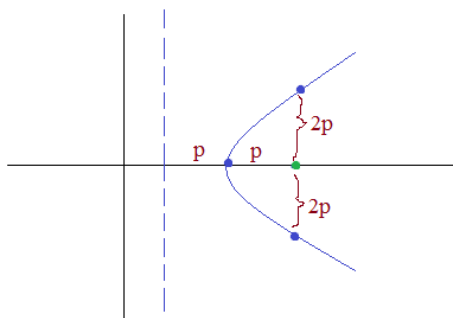
The "p" value is 2,
 so the focus is (-2, 6)

Latus Rectum

What is it? A line segment through the focus that is perpendicular to the line of symmetry (i.e. major axis), where the endpoints are on the parabola.

The length of the latus rectum is $4p$.

Graphing a parabola with details: using the lattice rectum



Distance from vertex to directrix: p

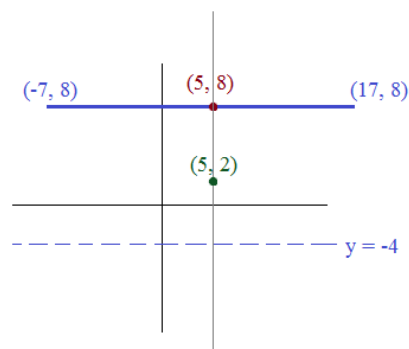
Distance from vertex to focus: p

Distance from focus to parabola (parallel to directrix): $2p$

Example: $(-7, 8)$ and $(17, 8)$ are the endpoints of the latus rectum.
The directrix is $y = -4$

Determine the equation of the parabola, graph, and check your solutions

- 1) Since $(-7, 8)$ and $(17, 8)$ are endpoints of the latus rectum,
we know the focus is the midpoint: $(5, 8)$
and, we know the parabola is vertical: $x^2 = 4py$



- 2) Since the focus is $(5, 8)$ and the directrix is $y = -4$,
the vertex is the midpoint! $(5, 2)$

- 3) $(x - 5)^2 = 4p(y - 2)$

p is the distance between the focus and vertex: 6
(or, it's $1/4$ the length of the latus rectum)

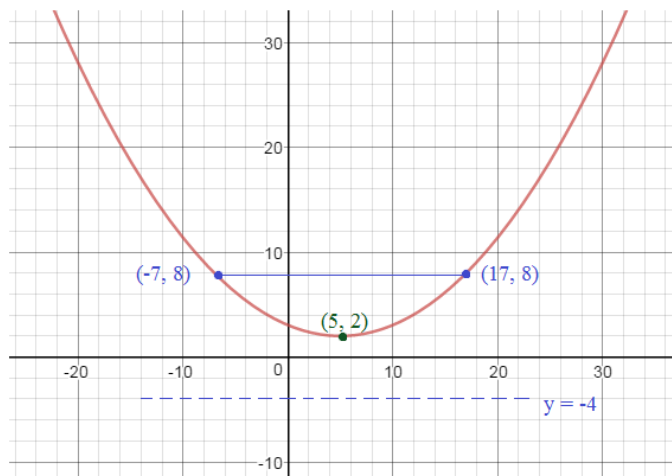
$$(x - 5)^2 = 24(y - 2)$$

- 4) To check our answers, we'll test 3 points...

$$(-7, 8) \text{ ----> } 144 = 24(6)$$

$$(17, 8) \text{ ----> } 144 = 24(6)$$

$$(5, 2) \text{ ----> } 0 = 24(0)$$



Example: Write the equation of a parabola with a vertex at the origin and directrix

$$x = -\frac{1}{8} \quad \text{Sketch the parabola.}$$

Since the directrix is a vertical line, we know the parabola opens left/right...

$$4px = y^2$$

$$p = \frac{1}{8}$$

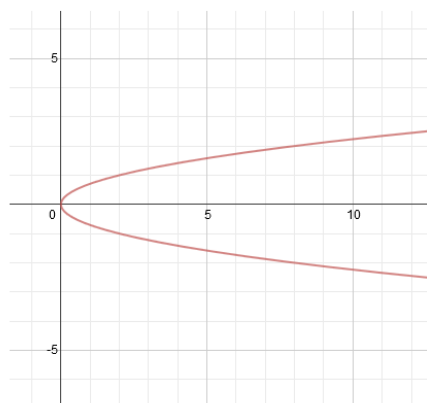
Since the directrix is on the left of the vertex $(0, 0)$, we know the parabola opens out to the right...

$$\frac{1}{2}x = y^2$$

Large sketch by plotting points..

To check the graph, plug in points:

$(8, 2)$ and $(8, -2)$ are points on the curve and fit in the equation!



Detailed magnified graph with vertex, directrix, focus, and latus rectum...

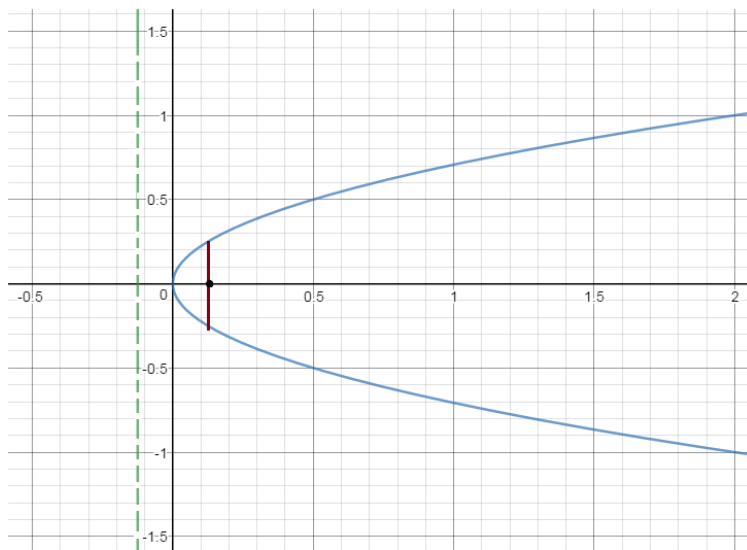
Vertex: $(0, 0)$

Directrix: $x = -\frac{1}{8}$

Focus: $(\frac{1}{8}, 0)$

Latus Rectum: segment from

$(\frac{1}{8}, \frac{1}{4})$ to $(\frac{1}{8}, -\frac{1}{4})$



Example: $4x^2 - 4x - 8y + 9 = 0$ Identify the vertex, focus, directrix, axis of symmetry, and latus rectum.

Separate the x's and y's, then complete the square

$$4(x^2 - x + 1/4) = 8y - 9 + 1$$

$$4(x - 1/2)^2 = 8(y - 1)$$

$$(x - 1/2)^2 = 2(y - 1)$$

Standard form of vertical parabola:

$$(x - h)^2 = 4p(y - k)$$

(h, k) is vertex

$$4p = 2 \implies p = \frac{1}{2}$$

Vertex: $(\frac{1}{2}, 1)$

Focus: $(\frac{1}{2}, \frac{3}{2})$

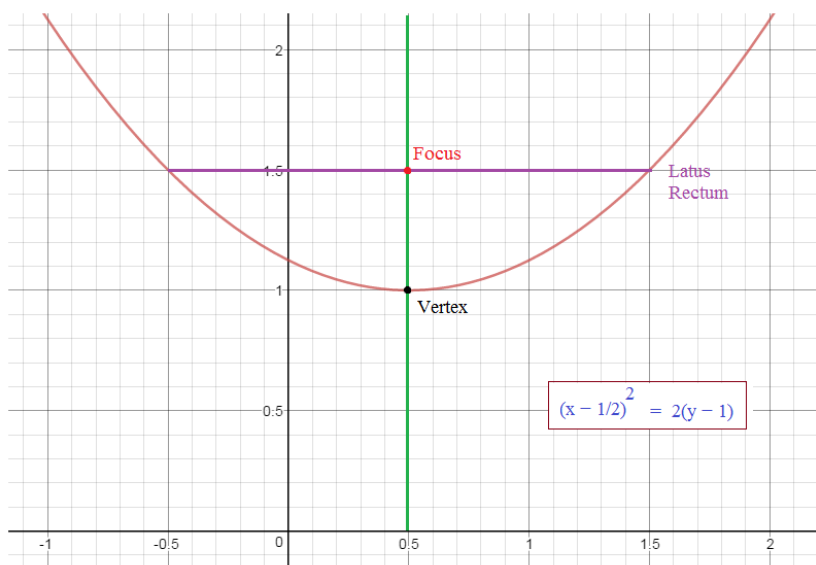
Directrix: $y = \frac{1}{2}$

Axis of Symmetry: $x = \frac{1}{2}$

Latus Rectum: Segment with endpoints

$(-\frac{1}{2}, \frac{3}{2})$ to $(\frac{3}{2}, \frac{3}{2})$

Axis of Symmetry



Parabolas Practice Quiz

I. Parts of a Parabola

Identify the focus, vertex, and directrix of each parabola. Find 2 other points on each figure.

A) $y = 2(x - 7)^2 + 1$

Vertex: _____

Focus: _____

Directrix: _____

2 points: _____

B) $y = x^2 - 11$

Vertex: _____

Focus: _____

Directrix: _____

2 points: _____

C) $x = -2(y + 6)^2 + 4$

Vertex: _____

Focus: _____

Directrix: _____

2 points: _____

D) $3x^2 - y - 24x + 55 = 0$

Vertex: _____

Focus: _____

Directrix: _____

2 points: _____

E) $y^2 - x - 10y + 28 = 0$

Vertex: _____

Focus: _____

Directrix: _____

2 points: _____

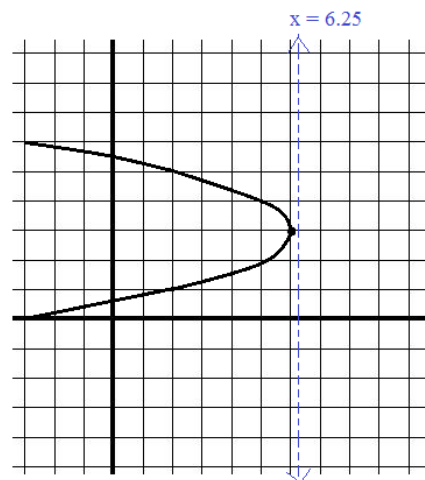
II. Equation of a Parabola

Determine the equation of each parabola from the given information.
(Express your answer in both *general form* and *standard form*)

- A) ("semi-axis length") $p = 2$
vertex: $(1, 2)$
x-intercept does not exist.

- B) Focus: $(3, 6)$
Directrix: $y = -2$

C)

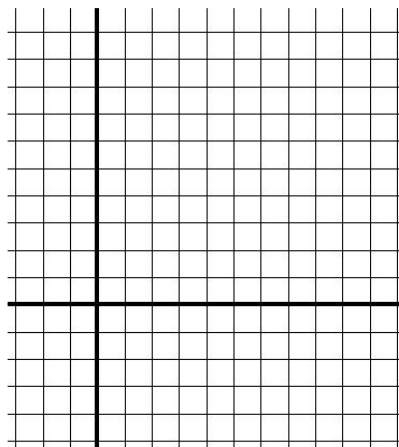


Parabola Practice Quiz

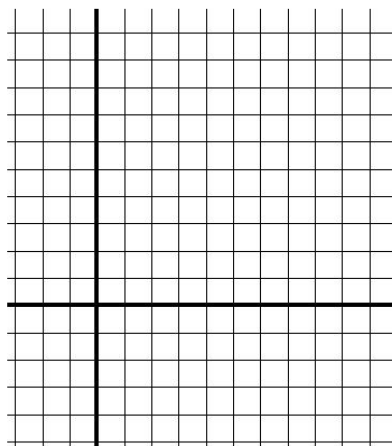
III. Graphing Parabolas

Graph the following parabolas; label the vertex, focus, directrix and intercepts.

A) $y = 2(x - 3)^2 - 4$



B) $-y^2 - x + 8y - 13 = 0$

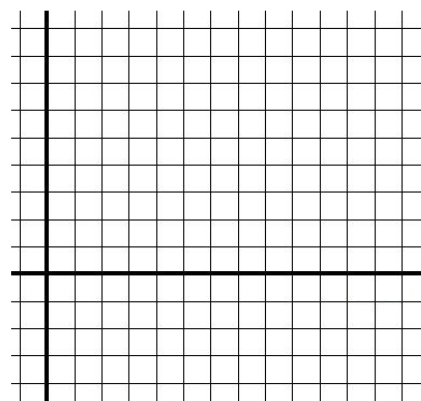


IV. Applications

A) Given Parabola: $x^2 - 10x - y + 23 = 0$

Line: $x + y = 9$

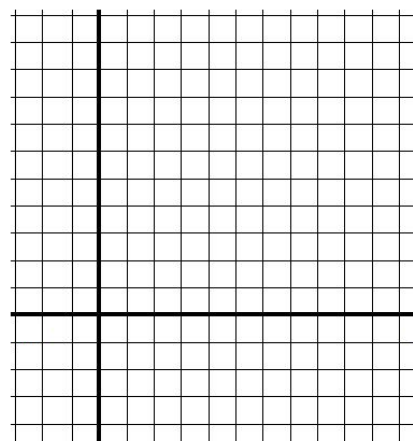
Determine the point(s) of intersection.
Solve algebraically and verify graphically.



B) Given Parabola P: $y = (x - 3)^2 + 3$

Find Parabola Q, where the vertex is the focus of P
and the focus is the vertex of P.

Sketch Parabolas P and Q.



Parabolas Practice Quiz

SOLUTIONS

Standard form: $y = a(x - h)^2 + k$
 where (h, k) is vertex; $p = \frac{1}{4a}$
 ("vertical parabola")

I. Parts of a Parabola

Identify the focus, vertex, and directrix of each parabola. Find 2 other points on each figure.

A) $y = 2(x - 7)^2 + 1$

$h = 7$
 $k = 1$

Vertex: $(7, 1)$

Focus: $(7, 1\frac{1}{8})$

Directrix: $y = 7/8$

2 points: $(0, 99)$

$(14, 99)$

focus: $(h, k + p)$
 directrix: $y = k - p$

if $x = 0$, then
 $y = 2(0 - 7)^2 + 1 = 99$

if $x = 14$, then
 $y = 2(14 - 7)^2 + 1 = 99$

B) $y = x^2 - 11$

$h = 0$
 $k = -11$

Vertex: $(0, -11)$

Focus: $(0, -43/4)$

Directrix: $y = -45/4$

2 points: $(\sqrt{11}, 0)$

$(-\sqrt{11}, 0)$

(parent function-
 quadratic shifted
 down 11 units)

$p = 1/4$

x-intercepts:
 $(\sqrt{11}, 0)$ $(-\sqrt{11}, 0)$

y-intercept:
 $(0, -11)$

C) $x = -2(y + 6)^2 + 4$ ("horizontal parabola")

Vertex: $(4, -6)$

Focus: $(3\frac{7}{8}, -6)$

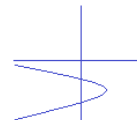
Directrix: $x = 4\frac{1}{8}$

2 points: $(-68, 0)$

$(-68, -12)$

$h = 4$
 $k = -6$
 $a = -2$

$p = -1/8$



if $y = 0$, then
 $x = -2(36) + 4 = -68$
 and
 if $y = -12$, then
 $x = -2(36) + 4 = -68$

D) $3x^2 - y - 24x + 55 = 0$

(complete the square and) put
 into standard/vertex form

Vertex: $(4, 7)$

Focus: $(4, 7\frac{1}{12})$

Directrix: $y = 6\frac{11}{12}$

2 points: $(0, 55)$

$(1, 34)$

$3(x^2 - 8x) + 55 = y$

$3(x^2 - 8x + 16) + 55 - 48 = y$

$y = 3(x - 4)^2 + 7$

$h = 4$
 $k = 7$
 $a = 3$
 $p = 1/12$

let $x = 0$, then
 $0 - y - 0 + 55 = 0$
 $y = 55$

let $x = 1$, then
 $3 - y - 24 + 55 = 0$
 $y = 34$

E) $y^2 - x - 10y + 28 = 0$

Vertex: $(3, 5)$

Focus: $(3\frac{1}{4}, 5)$

Directrix: $x = 2\frac{3}{4}$

2 points: $(28, 0)$

$(28, 10)$

(change to standard form)

$y^2 - 10y + 28 = x$

$y^2 - 10y + 25 + 28 - 25 = x$

$x = (y - 5)^2 + 3$

"horizontal parabola", so

$h = 3$ x -intercept: let $y = 0$,
 $k = 5$ then $x = 28$
 $a = 1$
 $p = 1/4$ then, let $y = 10$...
 and, $x = 28$

II. Equation of a Parabola

Determine the equation of each parabola from the given information.
 (Express your answer in both *general form* and *standard form*)

A) $p = 2$ "semi-axis length" or 1/2 focal
 vertex: $(1, 2)$ parameter
 x-intercept does not exist

since $p = 1/4a$ $2 = 1/4a$
 $a = 1/8$

$h = 1$
 $k = 2$

since x-intercept does not exist, the
 parabola must be "vertical".

$y = \frac{1}{8}(x - 1)^2 + 2$ standard form

expand and change to general form:

$y = \frac{1}{8}(x^2 - 2x + 1) + 2$

$\frac{x^2}{8} - \frac{x}{4} - y + \frac{17}{8} = 0$

general form

B) Focus: $(3, 6)$
 Directrix: $y = -2$

The vertex is between the focus and
 directrix:

$(3, 2)$ $h = 3$ $k = 2$

'p' is the distance from the vertex
 to the focus (or the directrix): 4 units

$p = 4$ (positive 4, because the focus
 is above the directrix)

$p = 1/4a$ so, $4 = 1/4a$ ---- $a = 1/16$

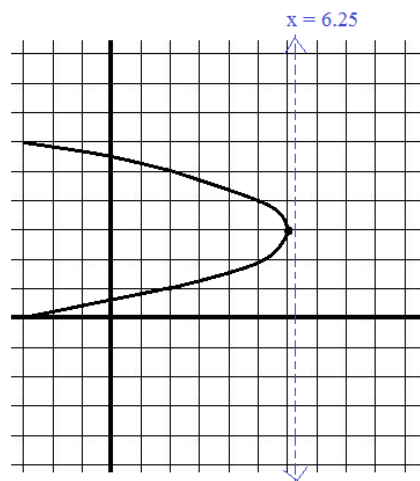
$y = \frac{1}{16}(x - 3)^2 + 2$ standard form

$y = \frac{x^2}{16} - \frac{6x}{16} + \frac{9}{16} + 2$

$\frac{x^2}{16} - \frac{3x}{8} + \frac{41}{16} - y = 0$

general form

C)



directrix: $x = 6.25$

vertex: $(6, 3)$

$p = -25$

then, focus is $(5.75, 3)$

$p = 1/4a$; therefore, $a = -1$

$x = -1(y - 3)^2 + 6$

$-y^2 + 6y - x - 3 = 0$

"Horizontal parabola
 that opens to the left"

III. Graphing Parabolas

Graph the following parabolas; label the vertex, focus, directrix and intercepts.

A) $y = 2(x - 3)^2 - 4$ ("vertical parabola opens up")

B) $-y^2 - x + 8y - 13 = 0$

complete the square: $y^2 - 8y + 13 = -x$

$$y^2 - 8y + 16 + 13 - 16 = -x$$

$$(y - 4)^2 - 3 = -x$$

$$x = -(y - 4)^2 + 3$$

("Horizontal parabola" opens to the left)

$$a = -1$$

$$\text{vertex: } (3, 4)$$

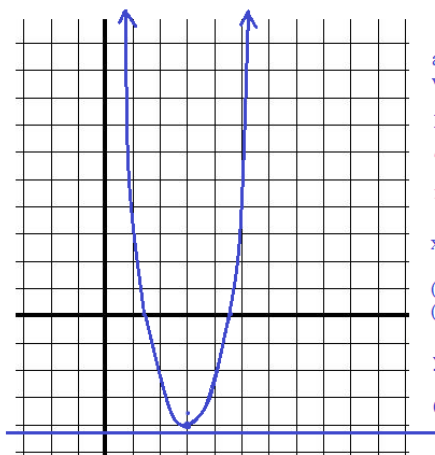
$$p = -1/4$$

$$\text{focus: } (2 \frac{3}{4}, 4)$$

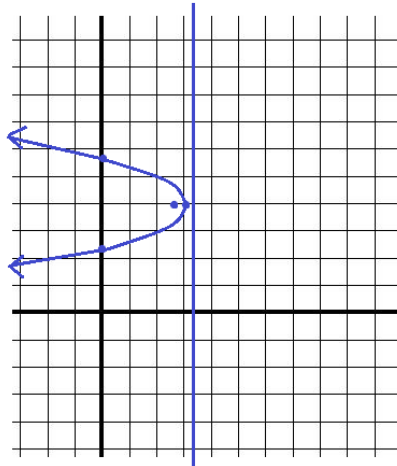
$$\text{directrix: } x = 3 \frac{1}{4}$$

$$\text{x-intercept: } (-13, 0)$$

$$\text{y-intercepts: } (0, 4 + \sqrt{3}), (0, 4 - \sqrt{3})$$



$a = 2$
 vertex $(h, k) = (3, -4)$
 $p = 1/4a = 1/8$
 directrix: $y = -4 \frac{1}{8}$
 focus: $(3, -3 \frac{7}{8})$
 x-intercepts:
 $(3 + \sqrt{2}, 0)$
 $(3 - \sqrt{2}, 0)$
 y-intercept:
 $(0, 14)$



IV. Applications

A) Given Parabola: $x^2 - 10x - y + 23 = 0$

Line: $x + y = 9$

Determine the point(s) of intersection.
 Solve algebraically and verify graphically.

$$x^2 - 10x - y + 23 = 0$$

$$y = 9 - x$$

use substitution:

$$x^2 - 10x - (9 - x) + 23 = 0$$

$$x^2 - 9x + 14 = 0$$

$$\text{if } x = 2, \text{ then } y = 7$$

$$(x - 2)(x - 7) = 0$$

$$\text{if } x = 7, \text{ then } y = 2$$

$$x = 2, 7$$

$$(2, 7) \quad (7, 2)$$

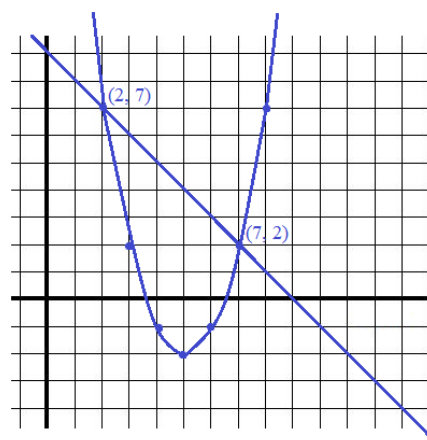
change parabola into standard form:

$$y = x^2 - 10x + 23$$

$$y = x^2 - 10x + 25 + 23 - 25$$

$$y = (x - 5)^2 - 2$$

vertex is $(5, -2)$
 and parabola opens up...



B) Given Parabola P: $y = (x - 3)^2 + 3$

Find Parabola Q, where the vertex is the focus of P
 and the focus is the vertex of P.

Sketch Parabolas P and Q.

$$\text{vertex P: } (3, 3)$$

$$\text{focus Q: } (3, 3)$$

$$\text{since } a = 1, p = 1/4$$

therefore,

$$\text{focus P: } (3, 3 \frac{1}{4})$$

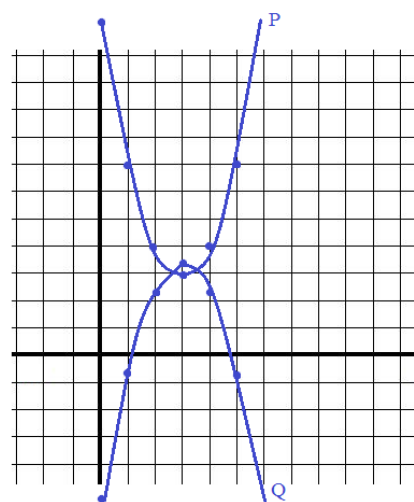
$$\text{vertex Q: } (3, 3 \frac{1}{4})$$

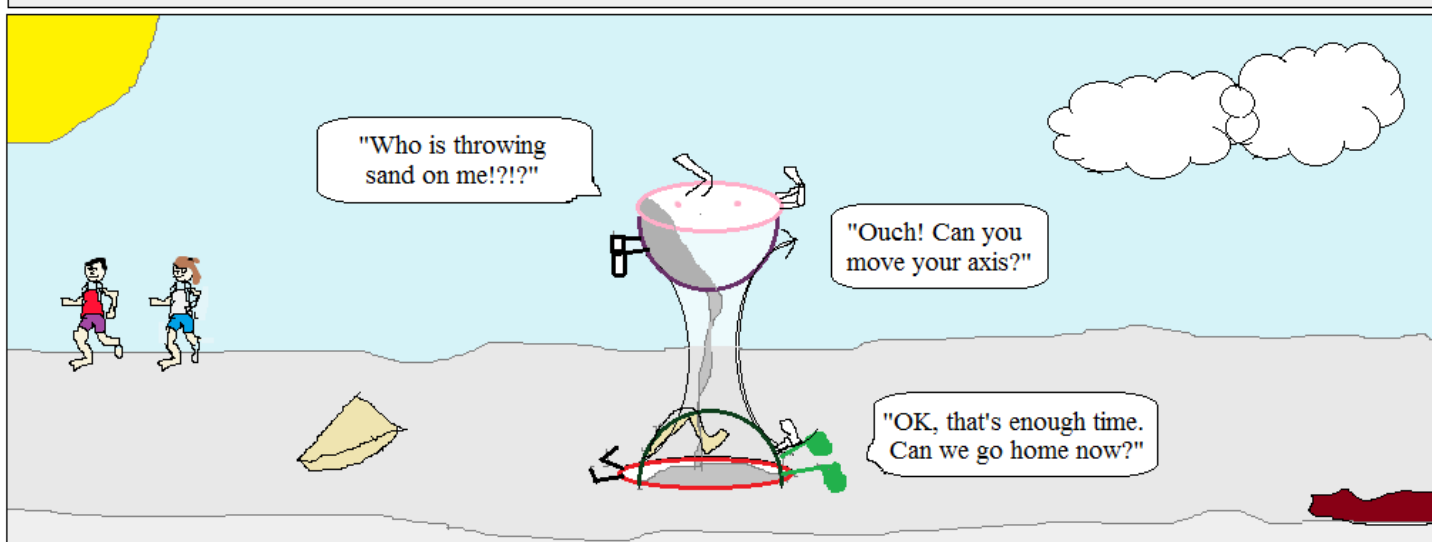
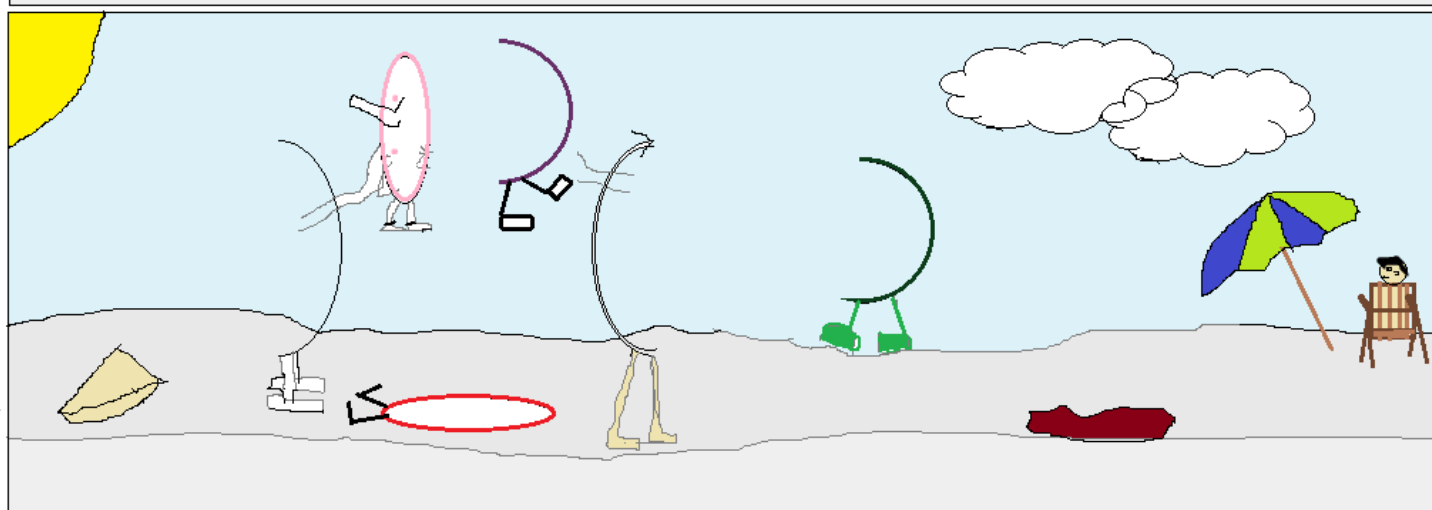
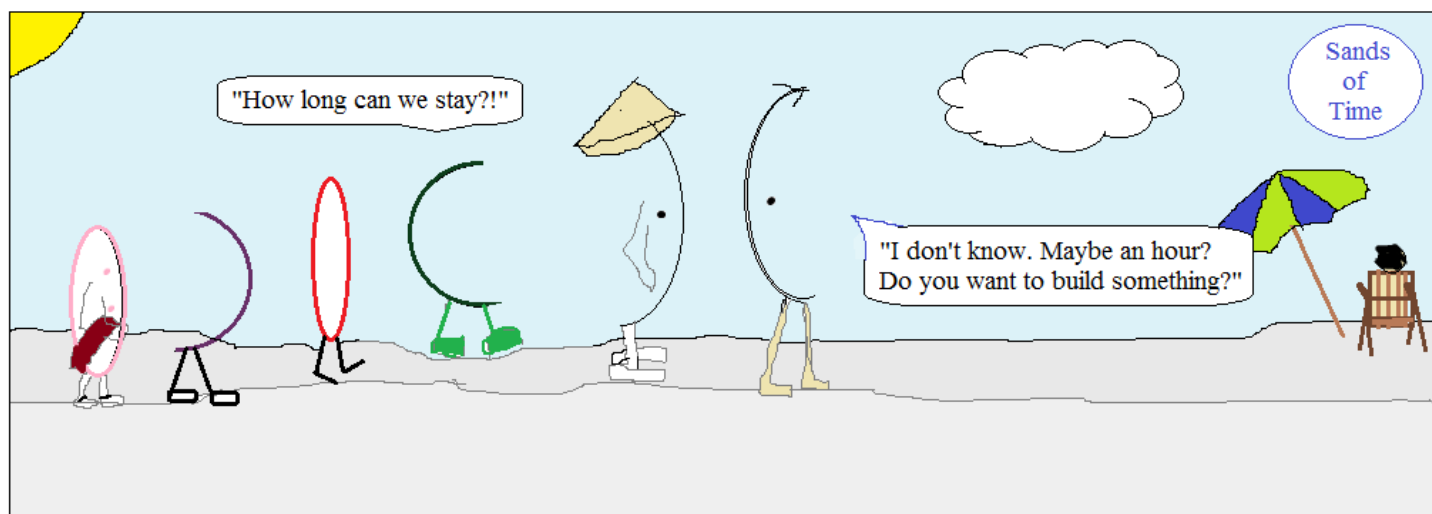
$$\text{directrix Q: } y = 3 \frac{1}{4}$$

$$\text{since } p = -1/4, a = -1$$

$$\text{Parabola P: } y = (x - 3)^2 + 3$$

$$\text{Parabola Q: } y = -(x - 3)^2 + 3 \frac{1}{4}$$



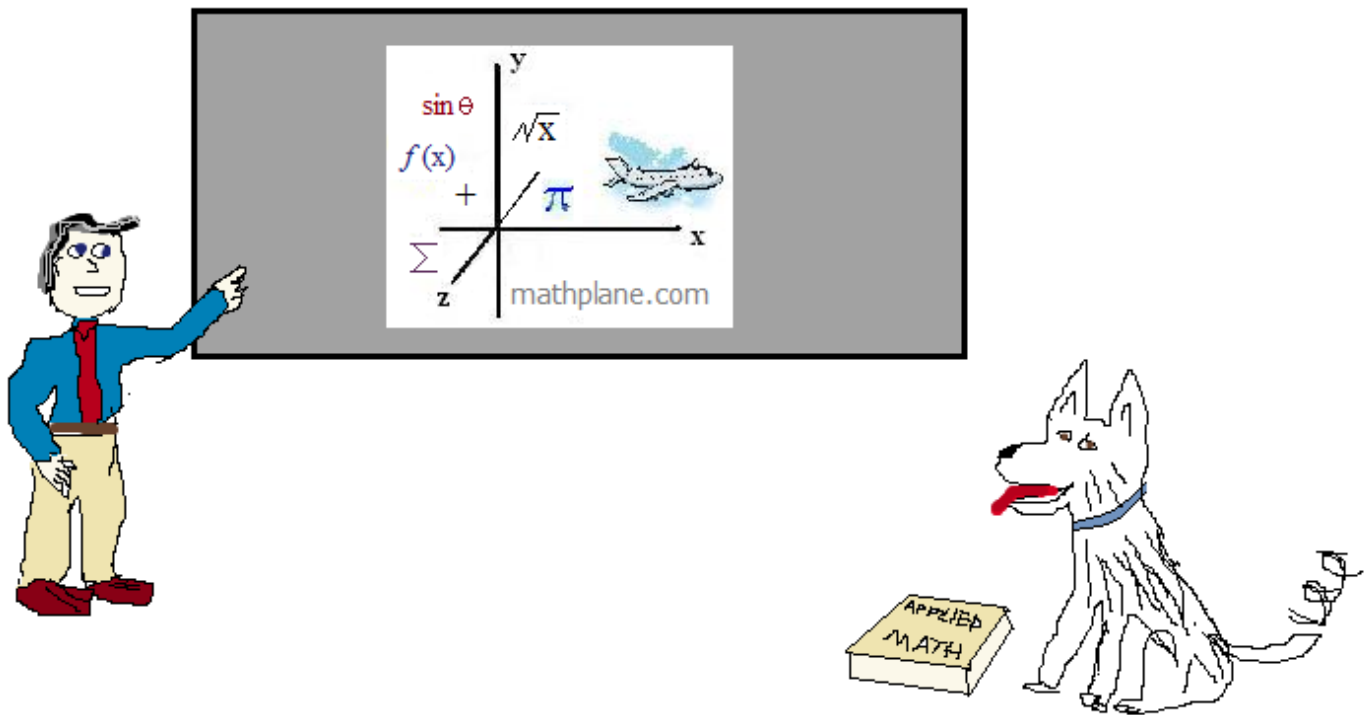


Harry Jr. and the remaining Balance of
Conicks spend an Hour at Glasse Beach...

LanceAF #151 (8-14-14)
mathplane.com

Thanks for downloading the packet. (Hope it helped!). We appreciate the support. Proceeds go to site maintenance (and, treats for Norway the Husky!).

Cheers...



Also, at Facebook, Google+, TeachersPayTeachers, and Pinterest

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